

Space-time deterministic Graph Rewriting

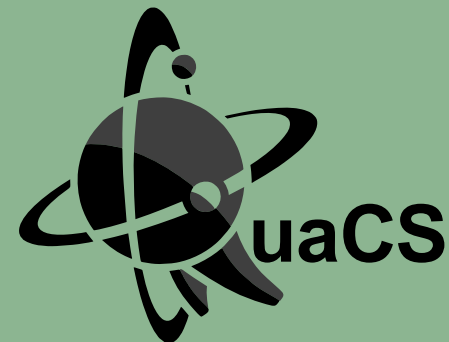
Pablo Arrighi, Marin Costes, Gilles Dowek, Luidnel Maignan

école _____
normale _____
supérieure _____
paris-saclay _____

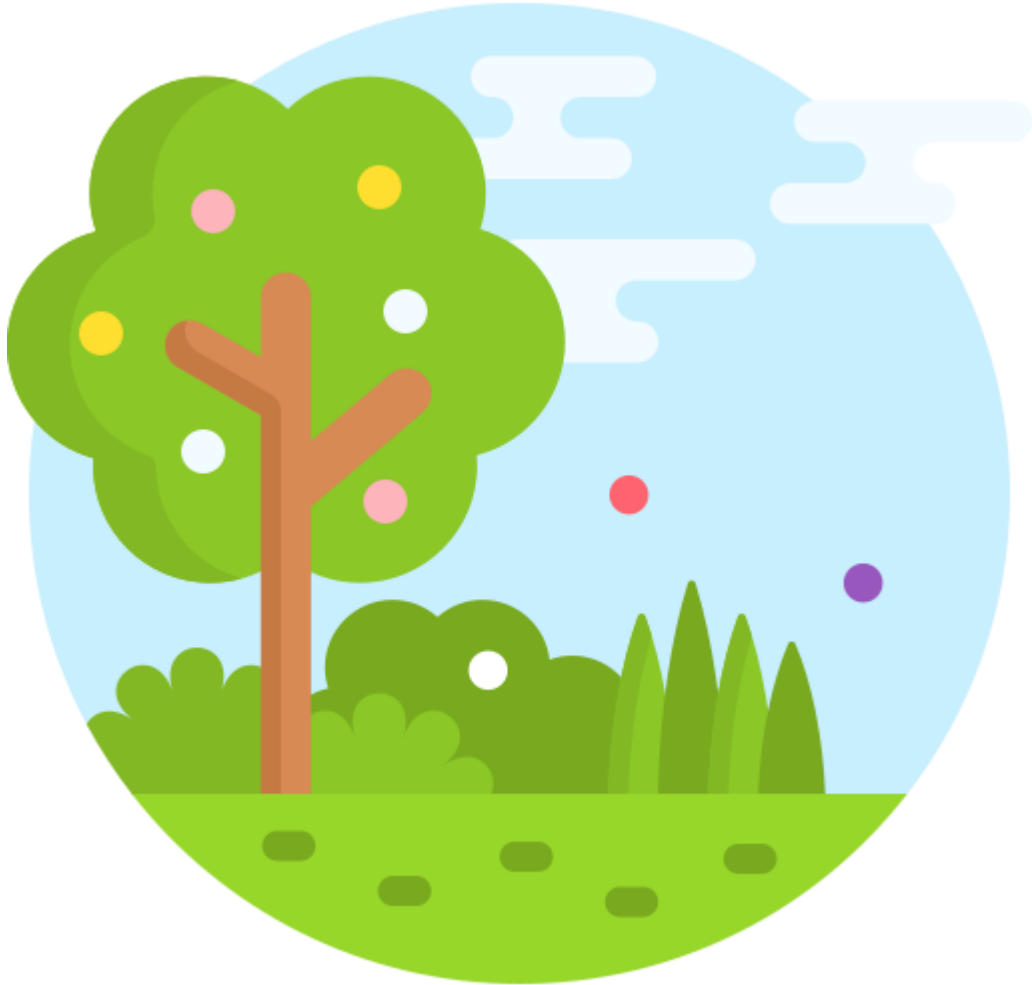
université
PARIS-SACLAY



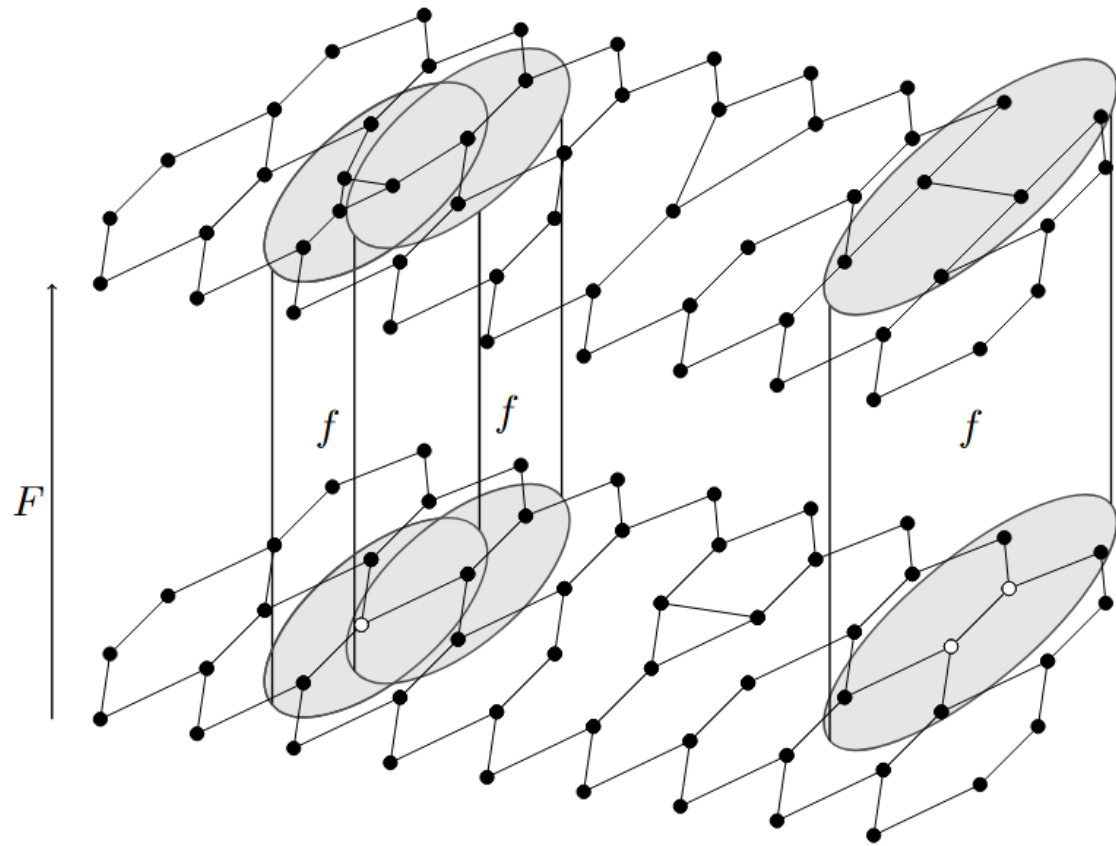
Laboratoire
Méthodes
Formelles



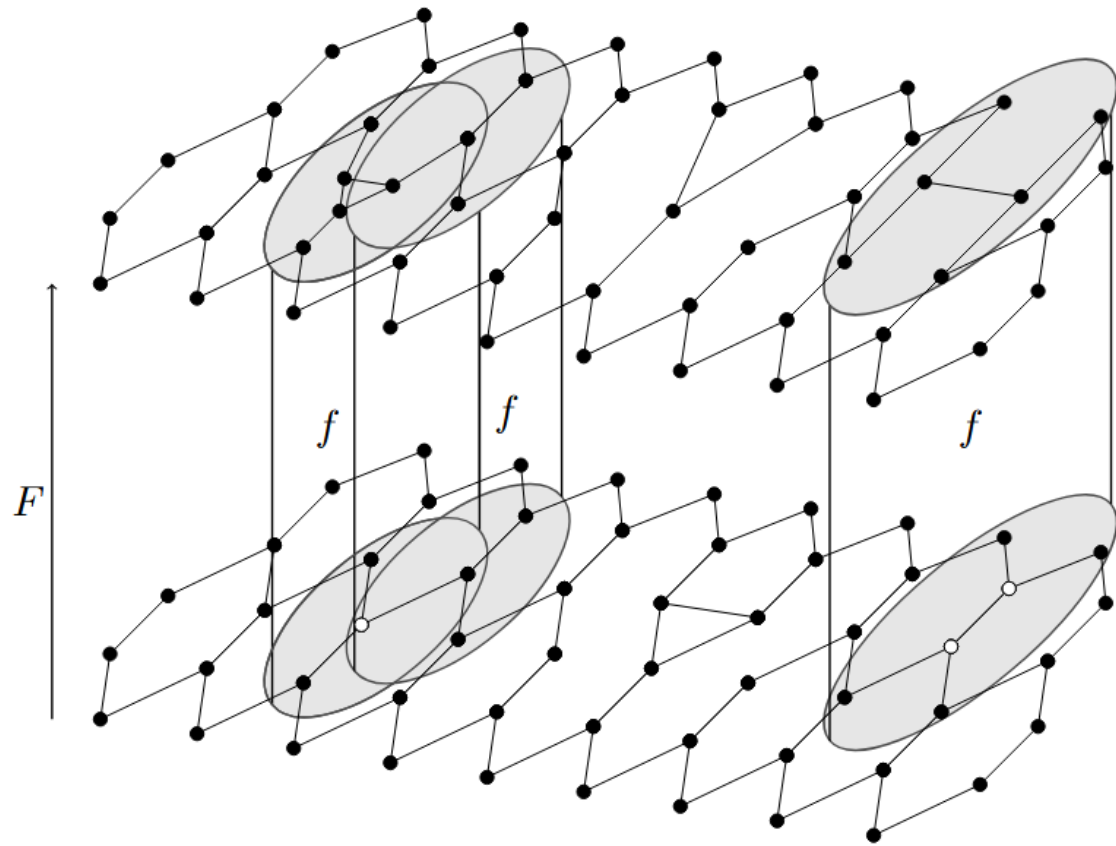
Simulating physics



Simulating physics

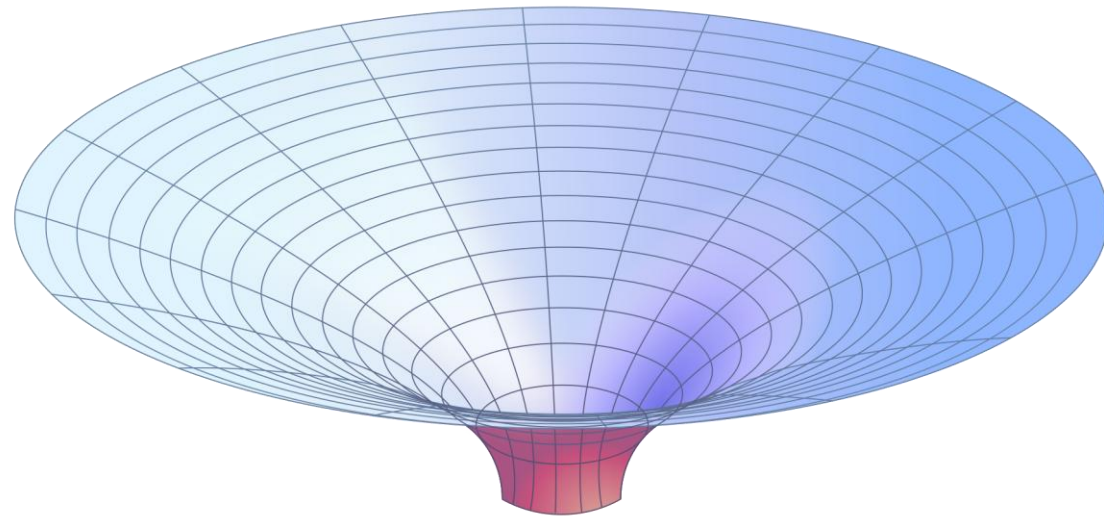


Simulating physics



\neq

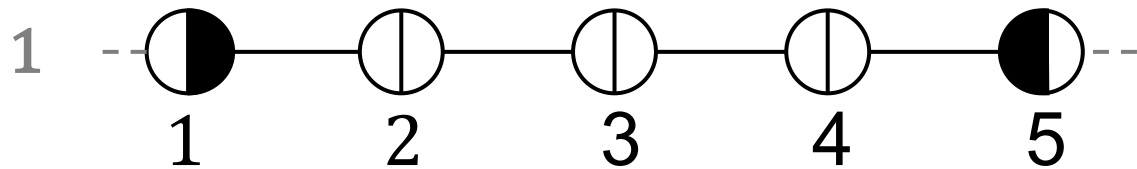
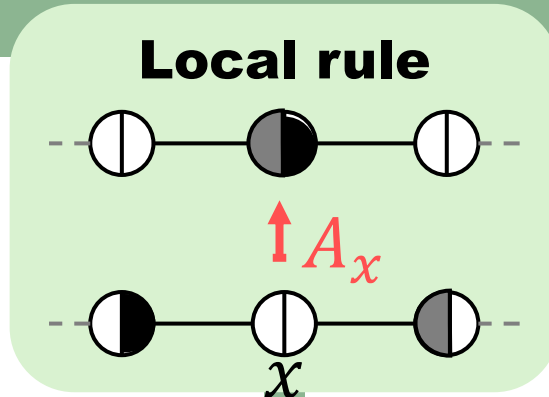
No global clock in physics !



Dynamical systems and rewriting

**Dynamical system
(Synchronous)**

time



Dynamical systems and rewriting

**Dynamical system
(Synchronous)**

Local rule

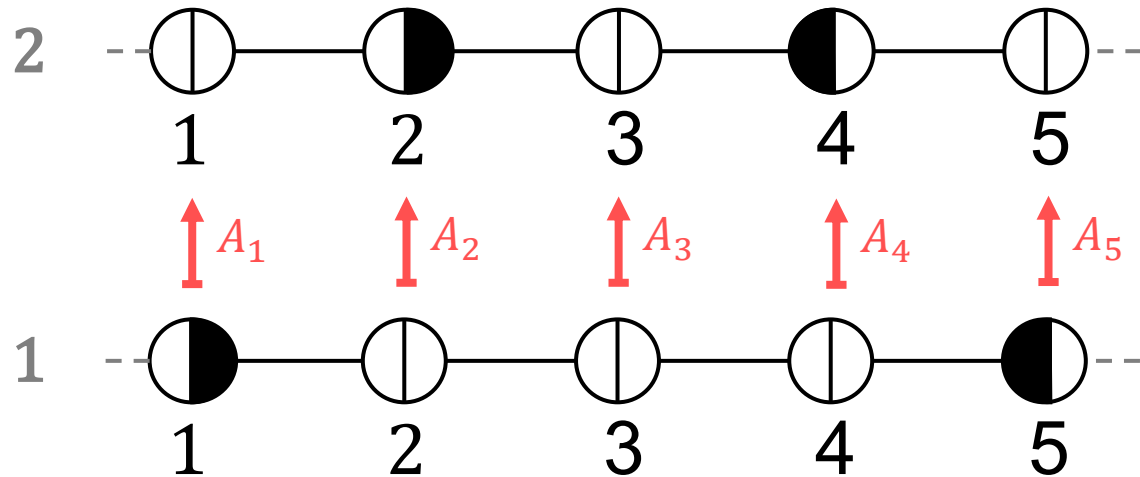


$\uparrow A_x$



x

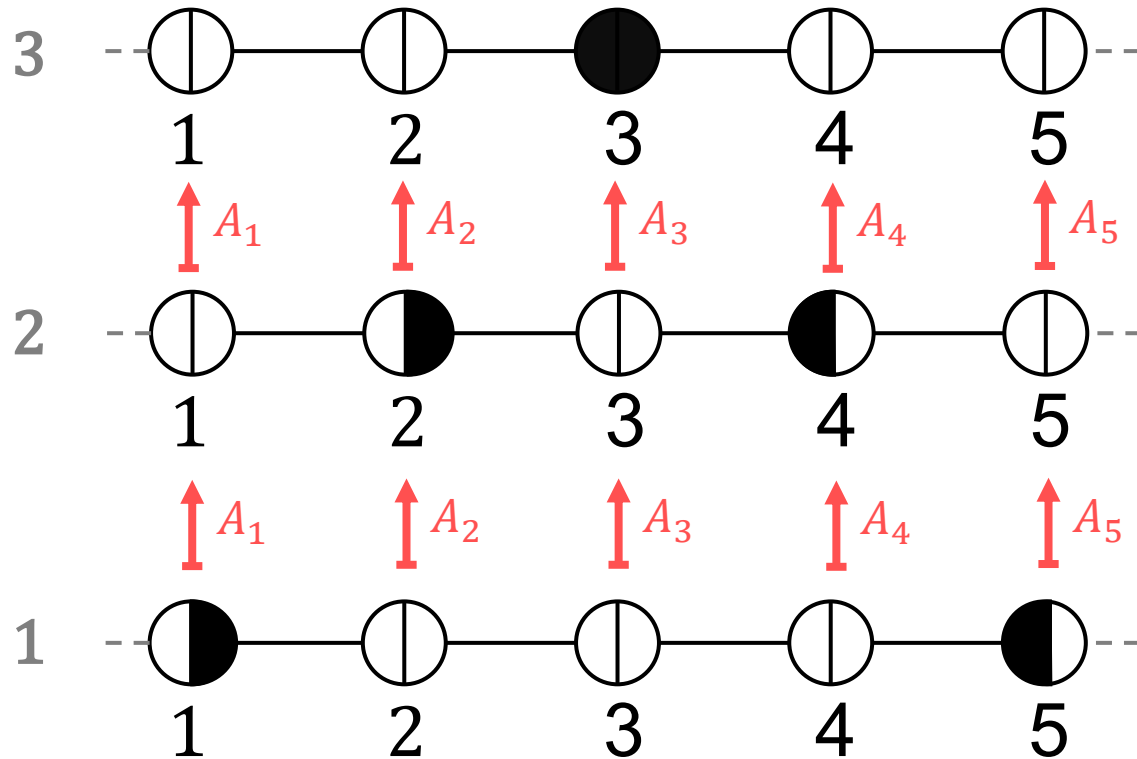
time



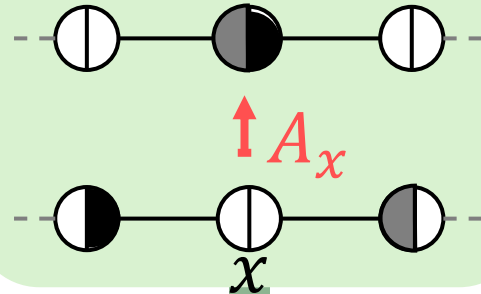
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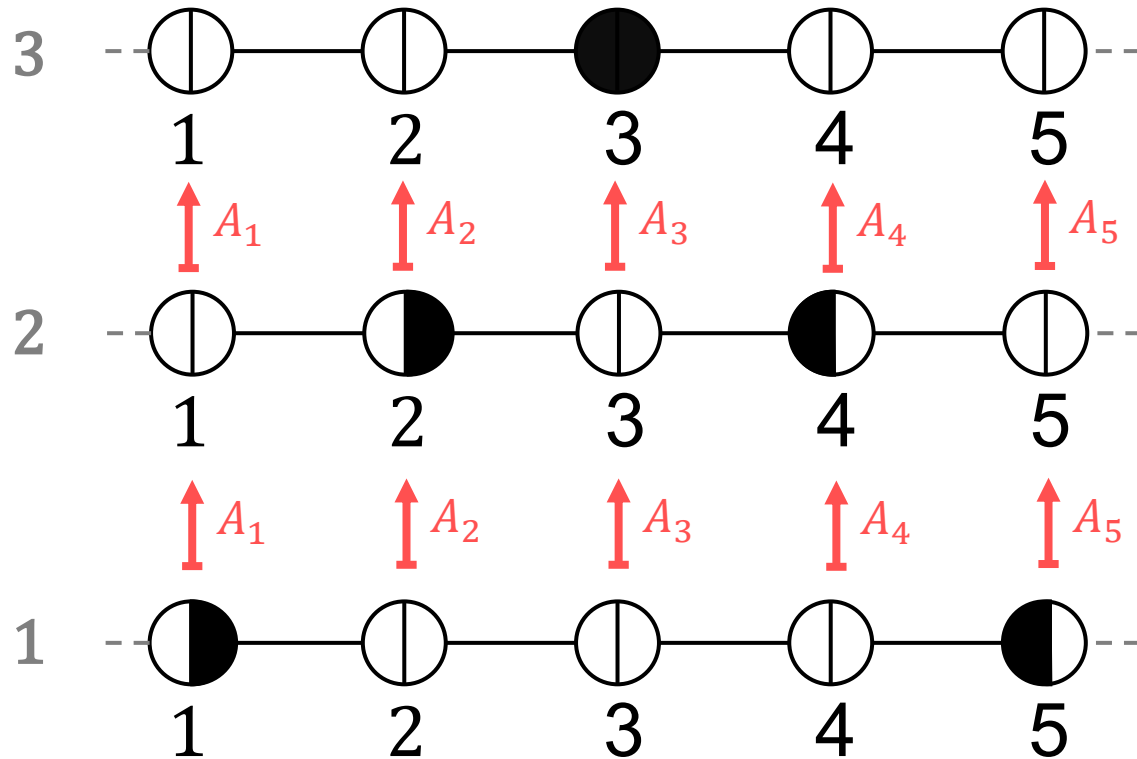
Local rule



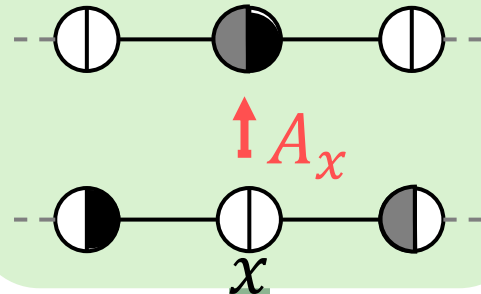
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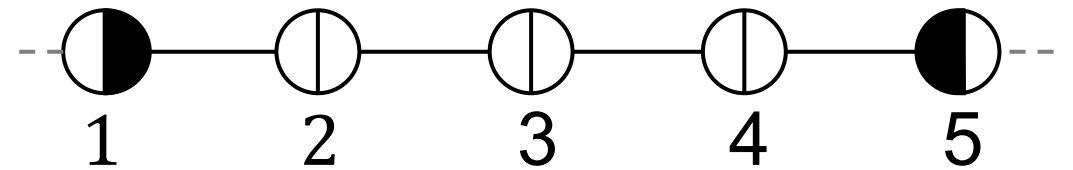


Local rule



**Graph rewriting
(Asynchronous)**

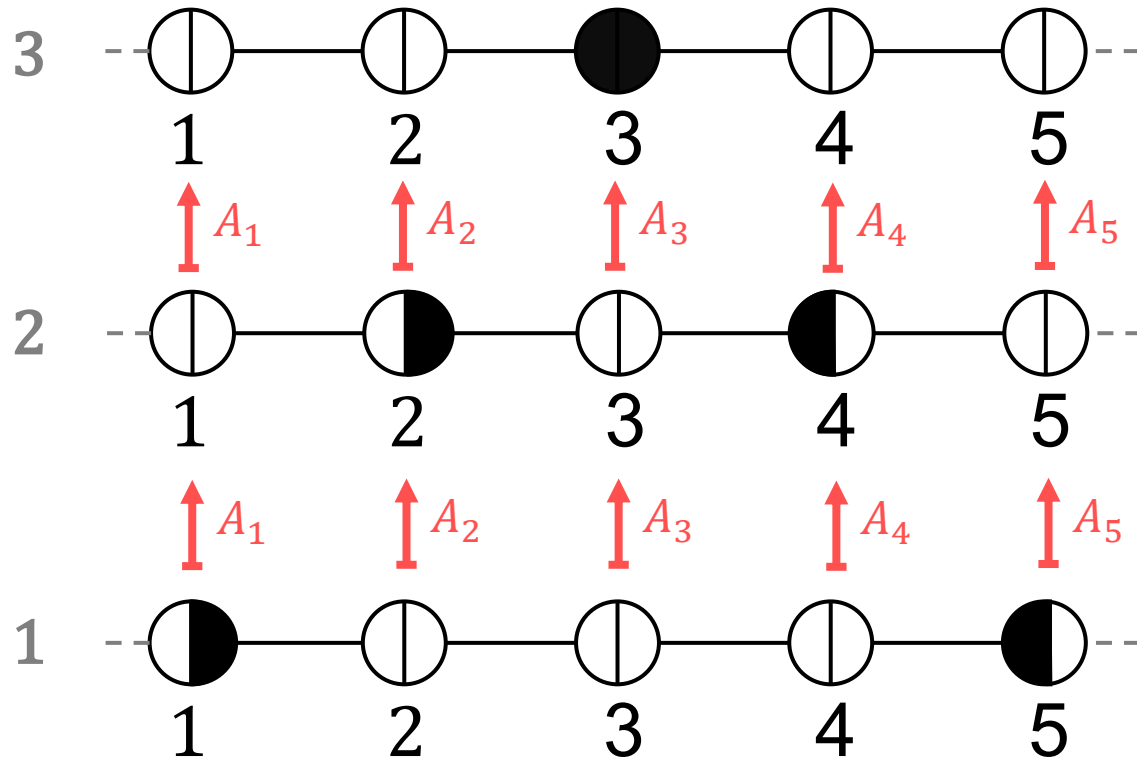
G



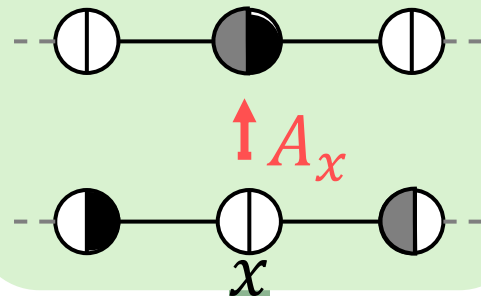
Dynamical systems and rewriting

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time

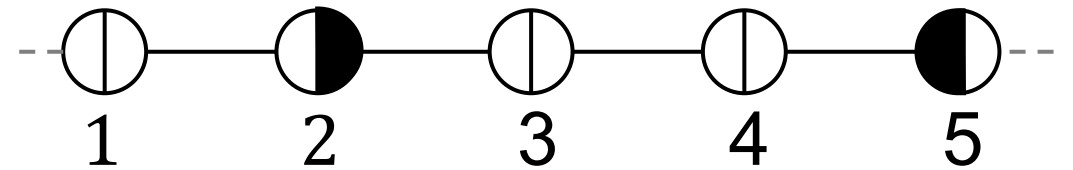


Local rule



**Graph rewriting
(Asynchronous)**

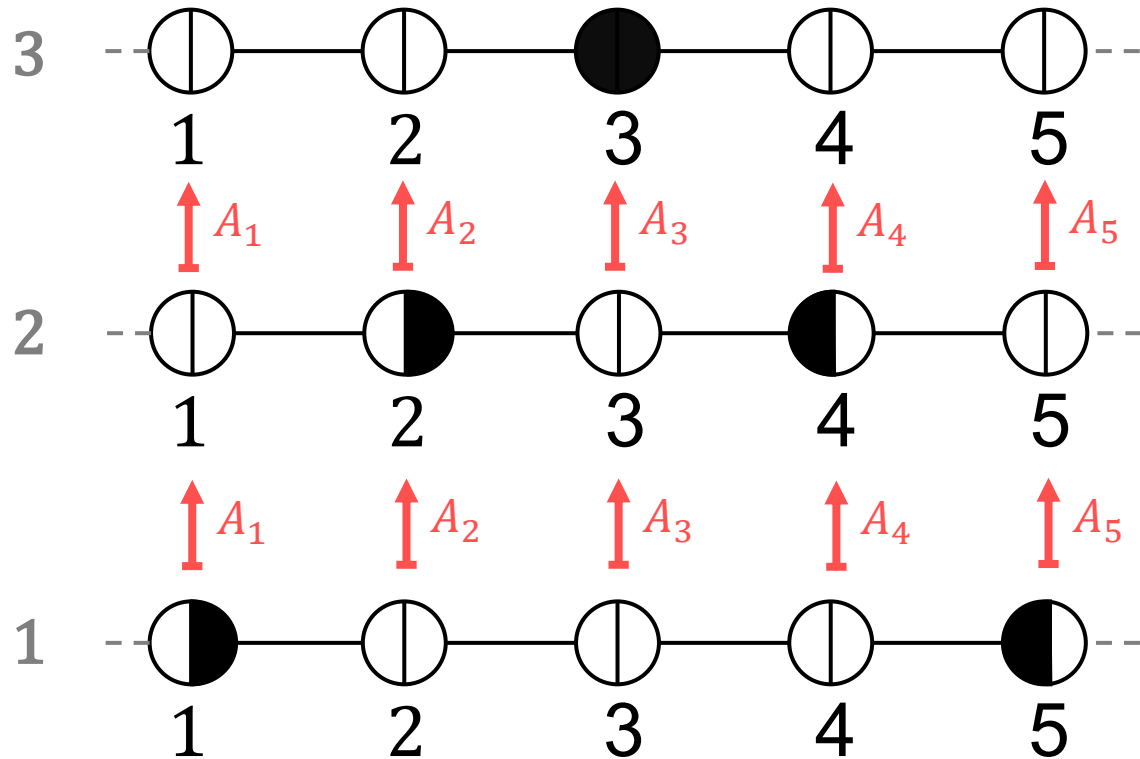
$A_2 G$



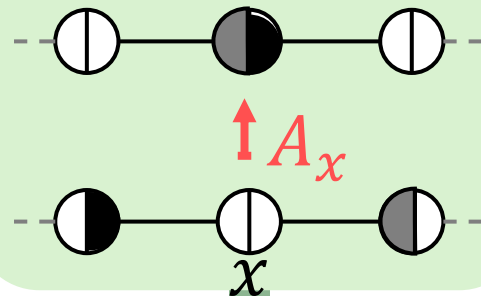
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time

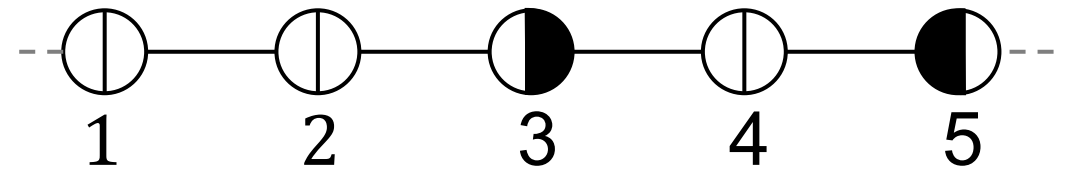


Local rule



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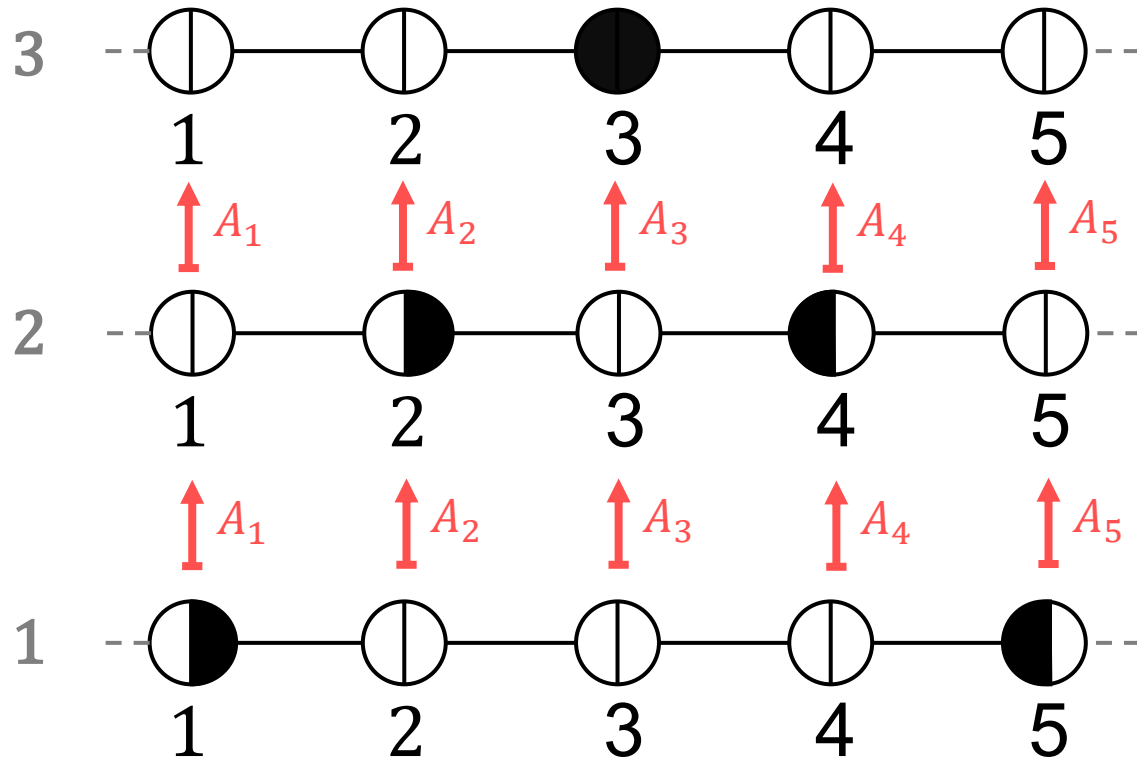
$A_3 A_2 G$



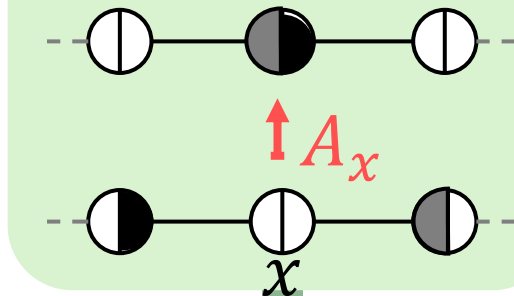
Dynamical systems and rewriting

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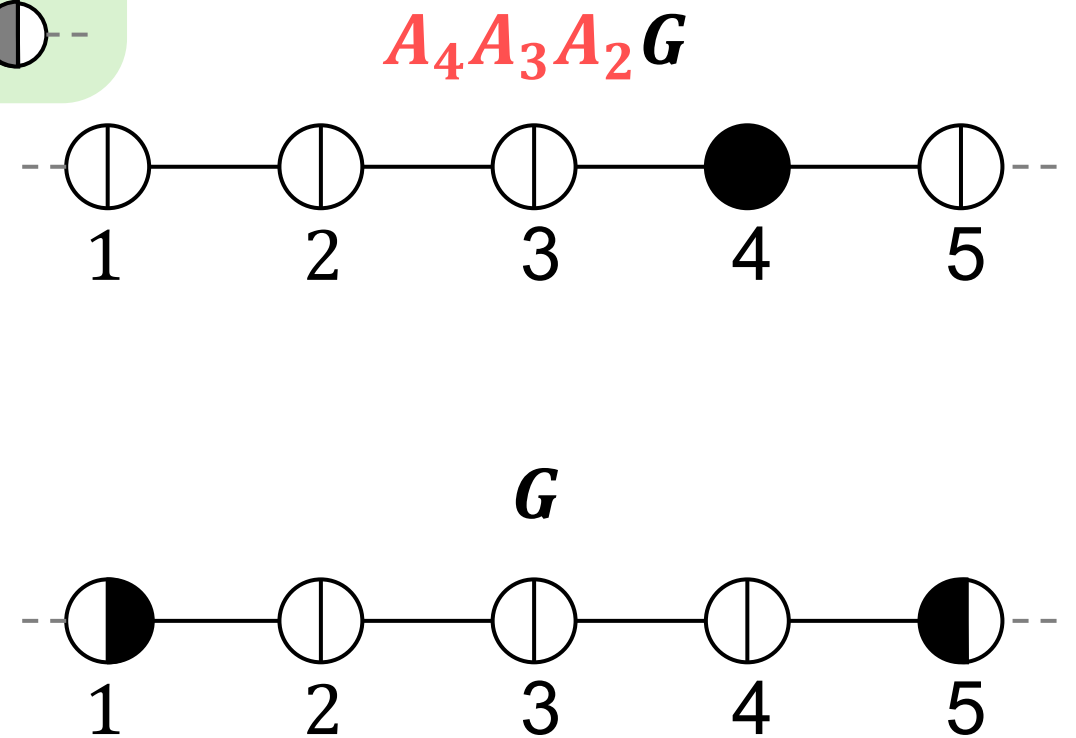
time



Local rule



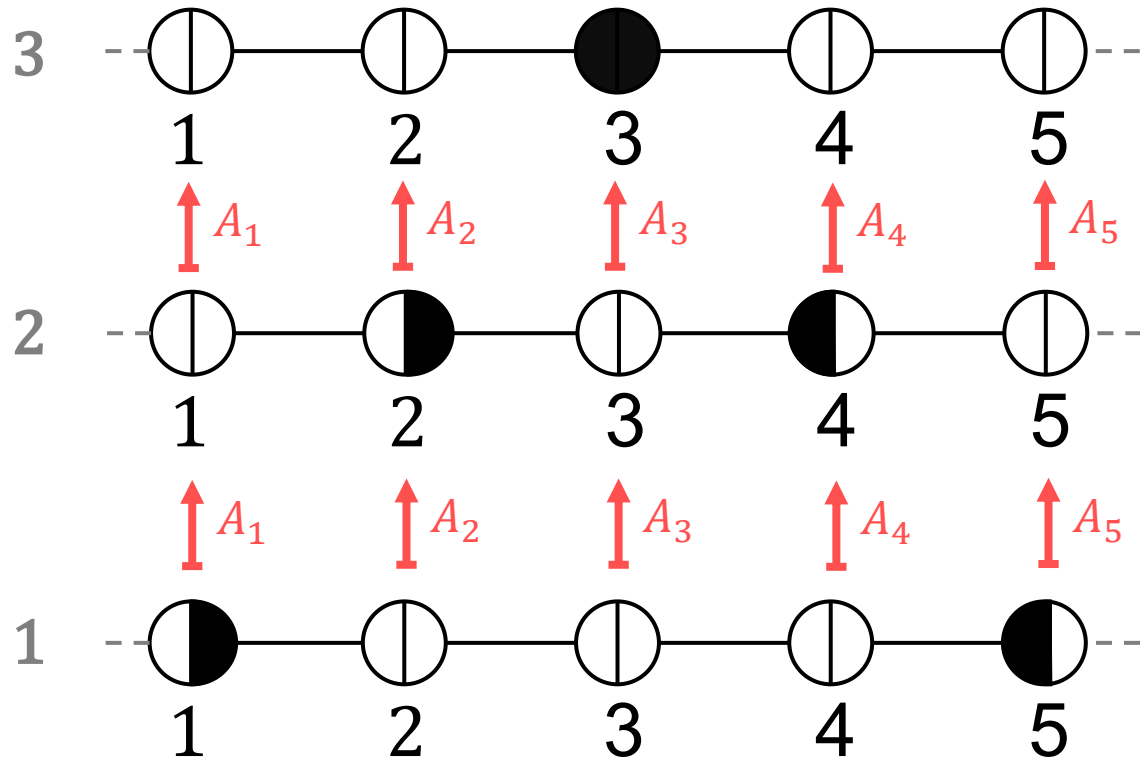
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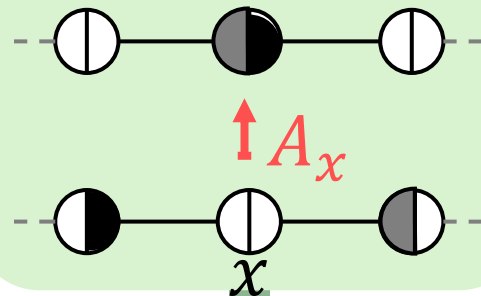
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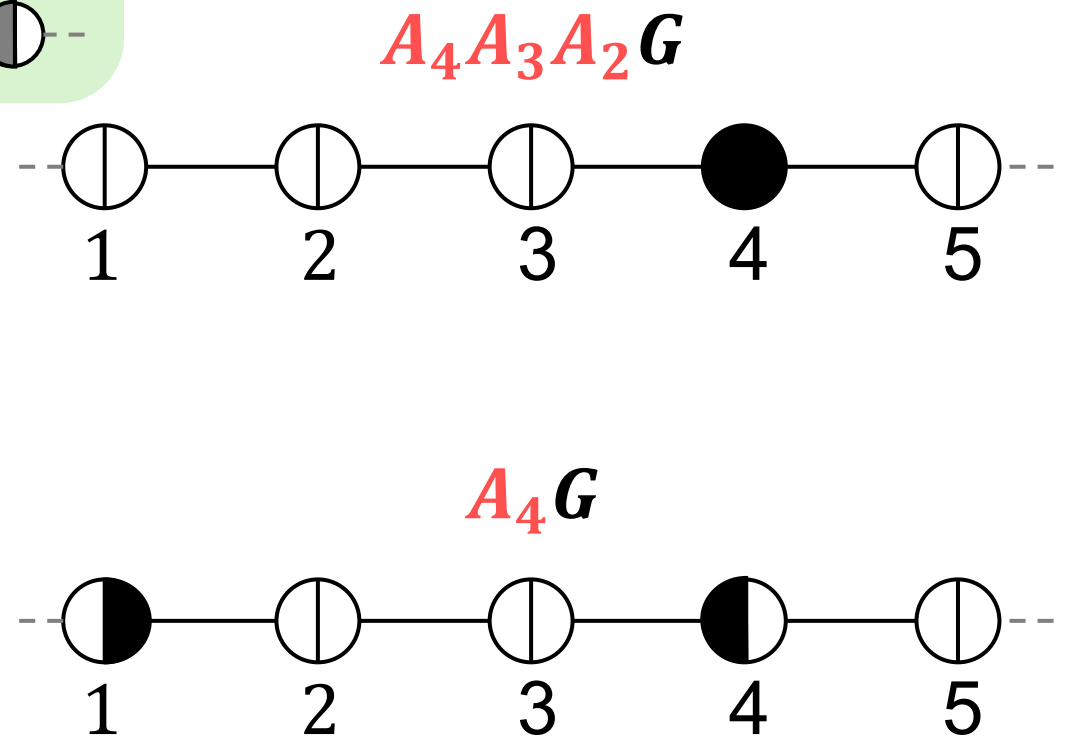
time



Local rule



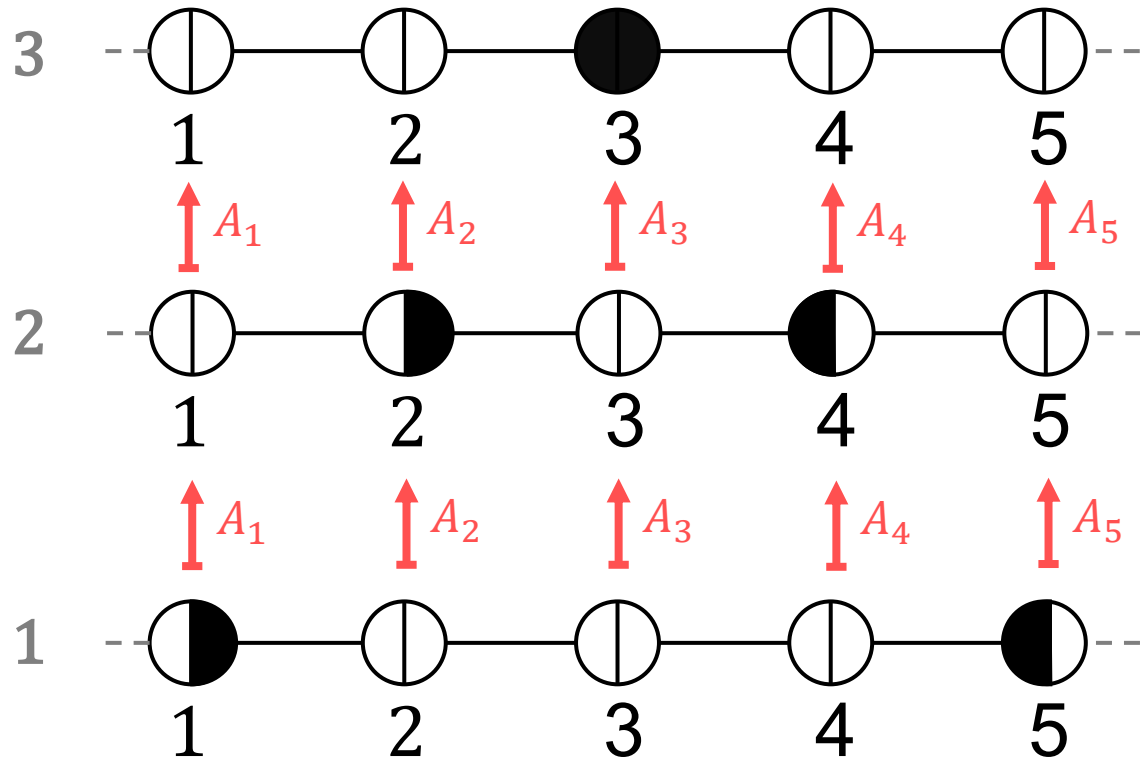
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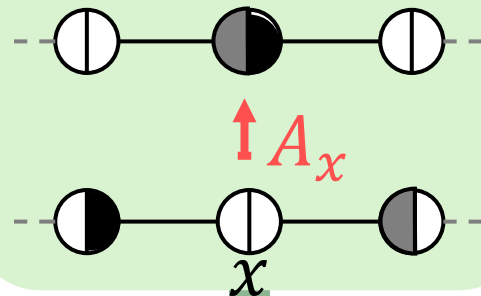
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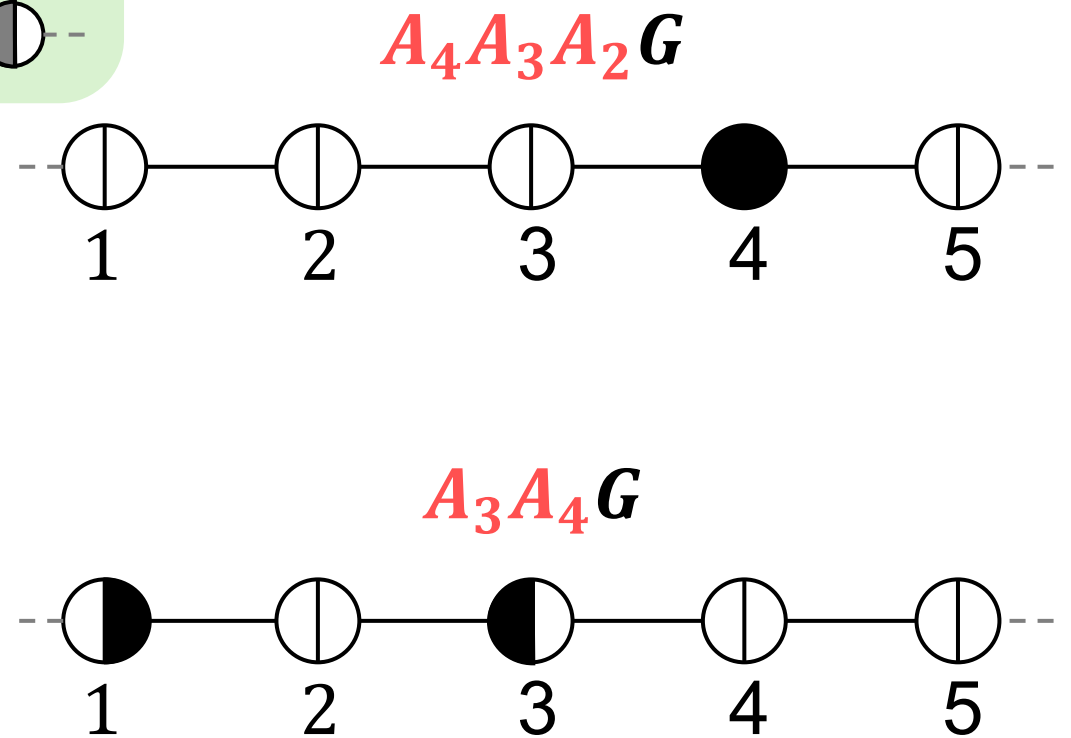
time



Local rule



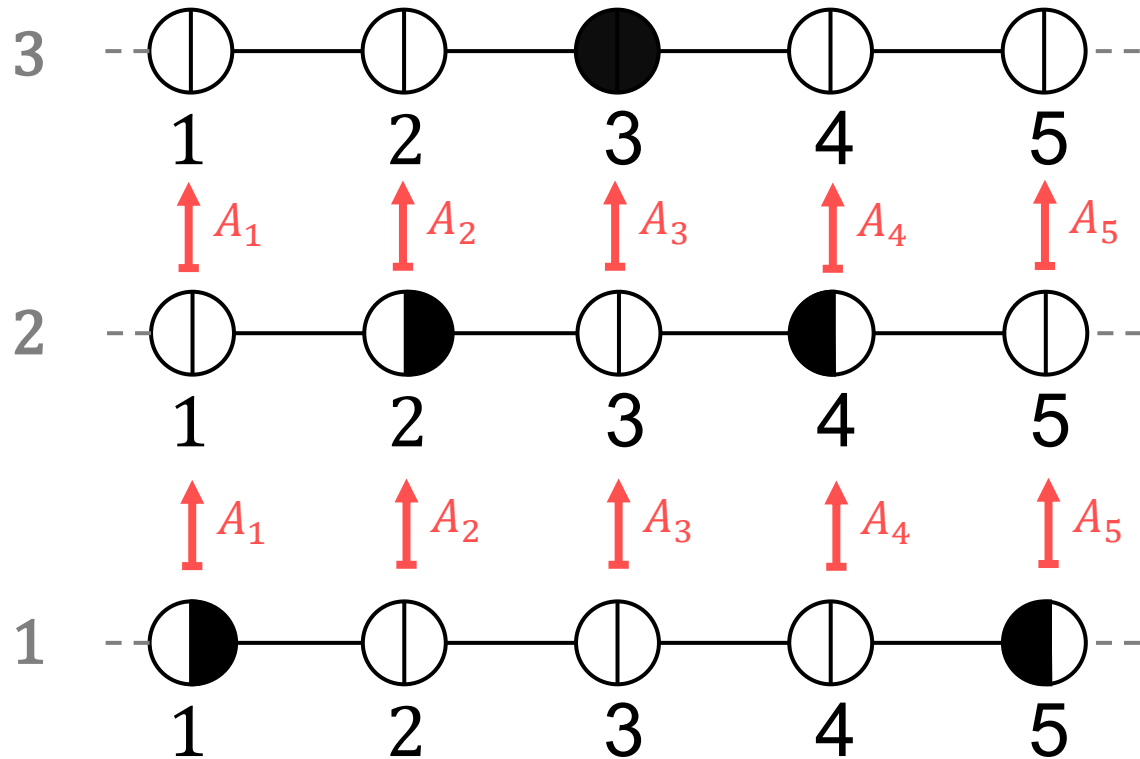
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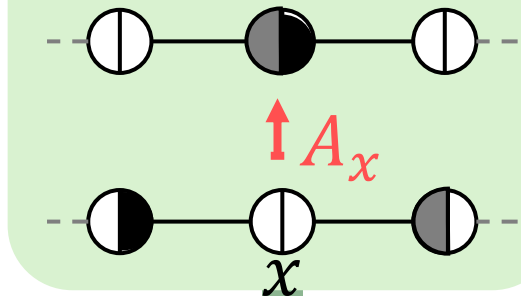
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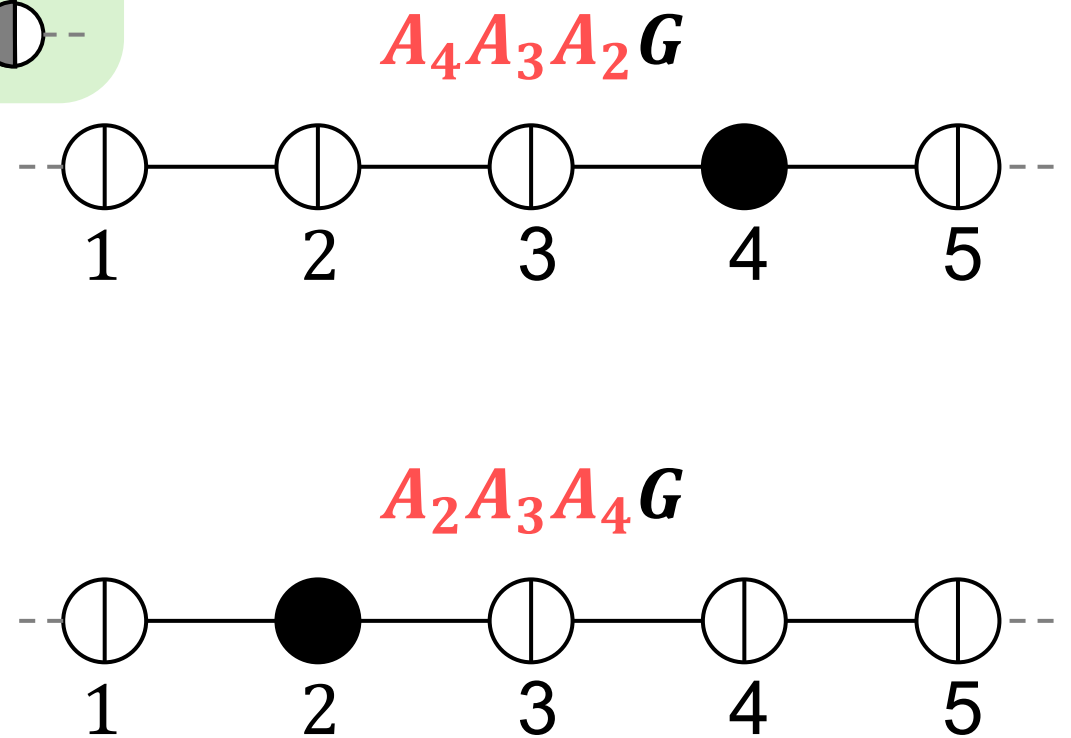
time



Local rule



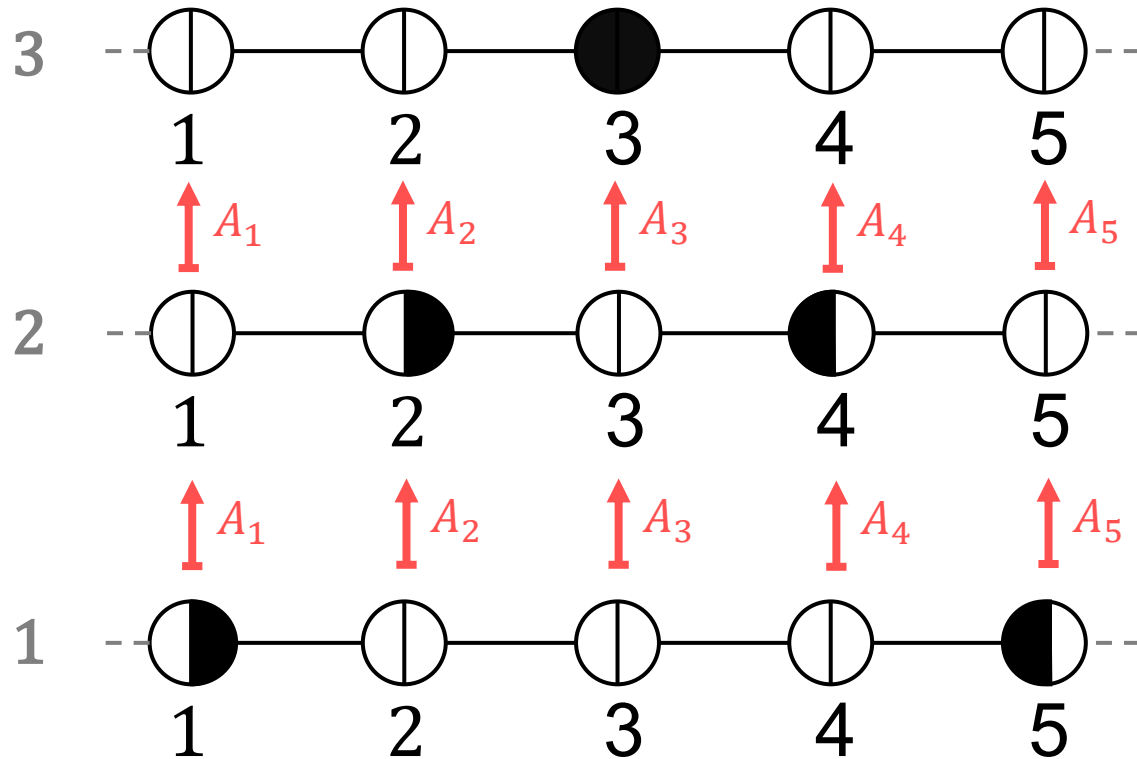
**Graph rewriting
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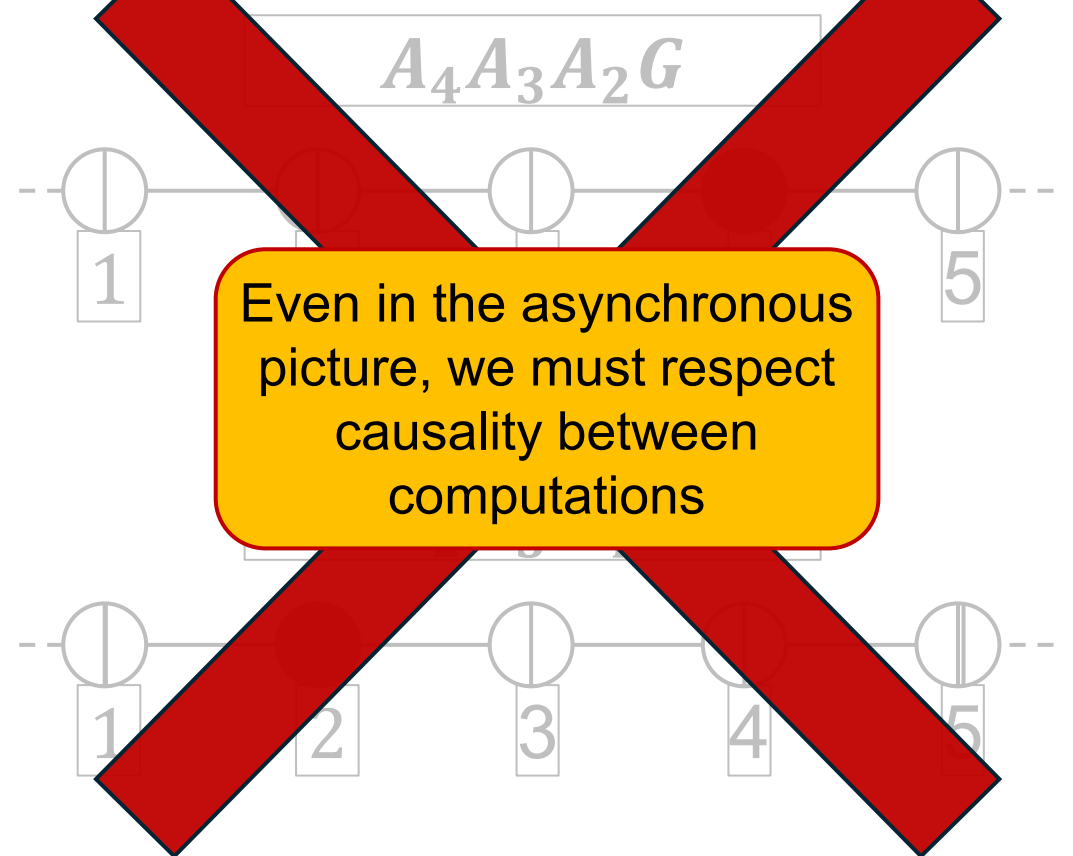
Dynamical systems and rewriting

Dynamical system (Synchronous)

time



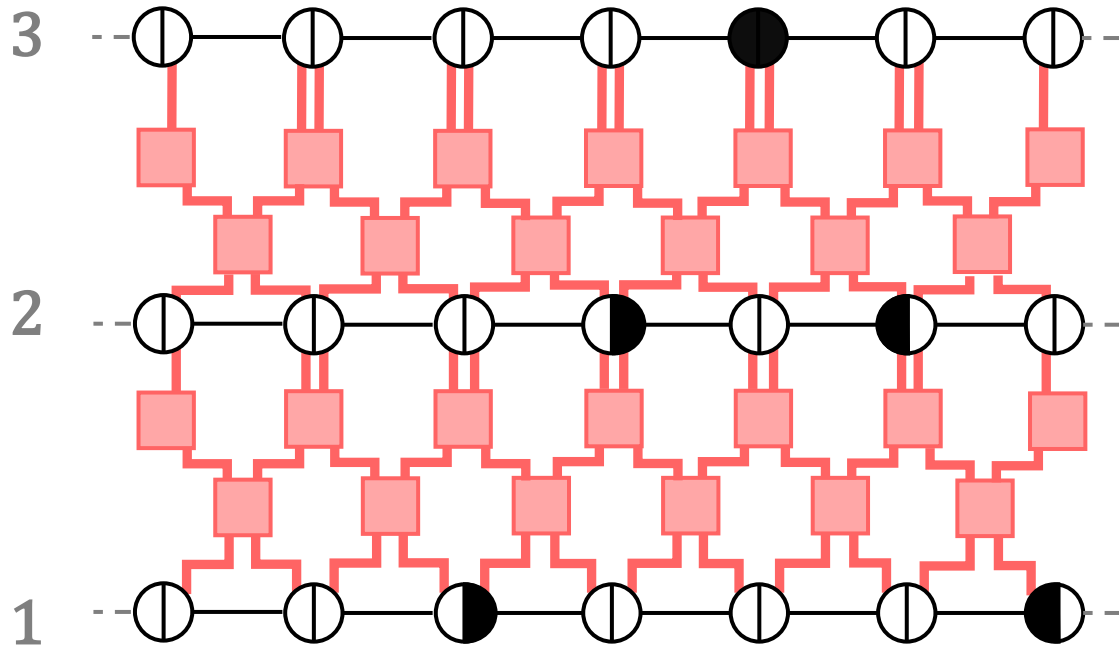
Naive asynchronous counterpart



Dynamical systems and rewriting

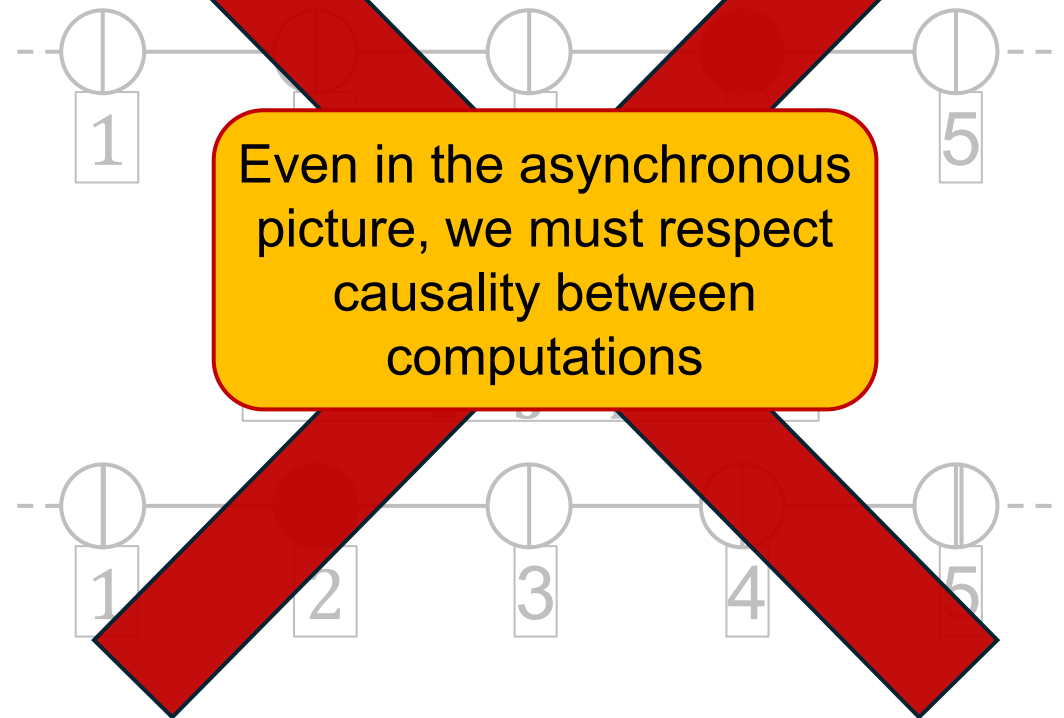
Causal structure

time



Naive asynchronous counterpart

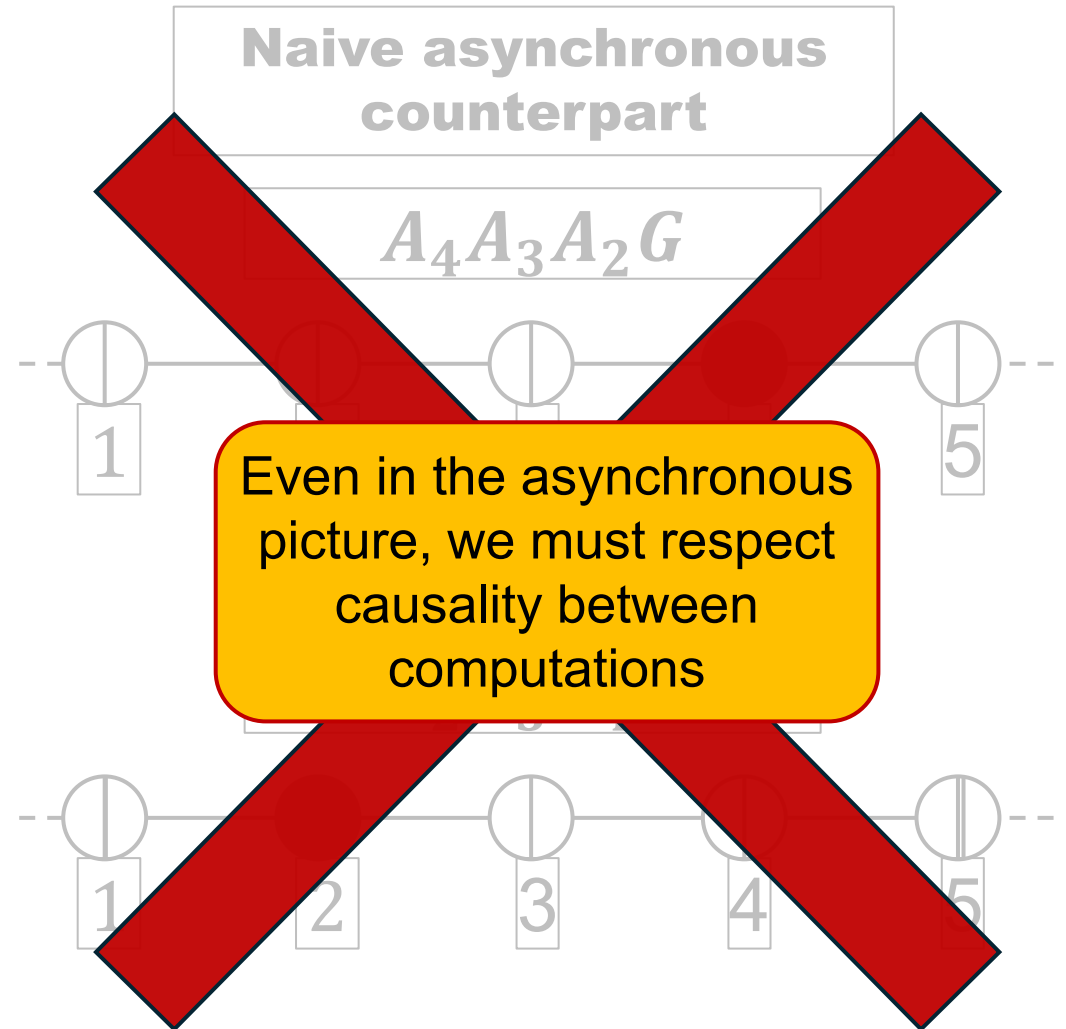
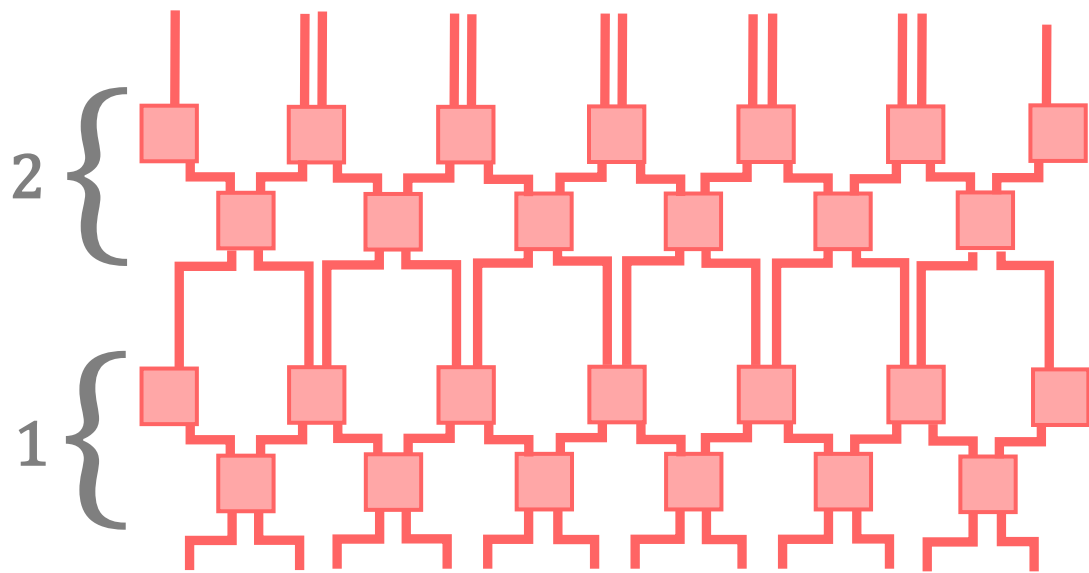
$A_4 A_3 A_2 G$



Dynamical systems and rewriting

Causal structure

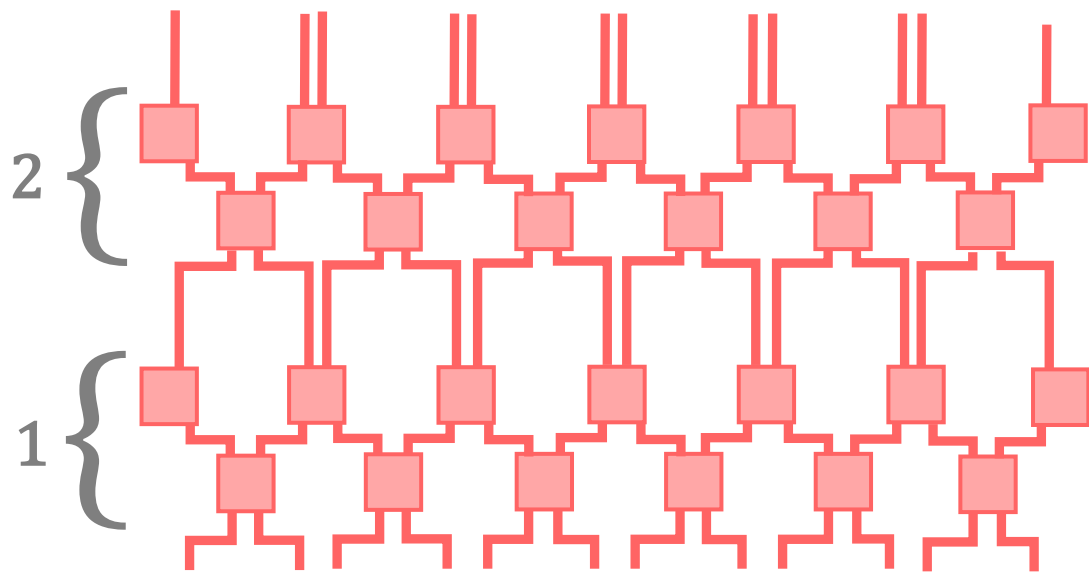
time



Dynamical systems and rewriting

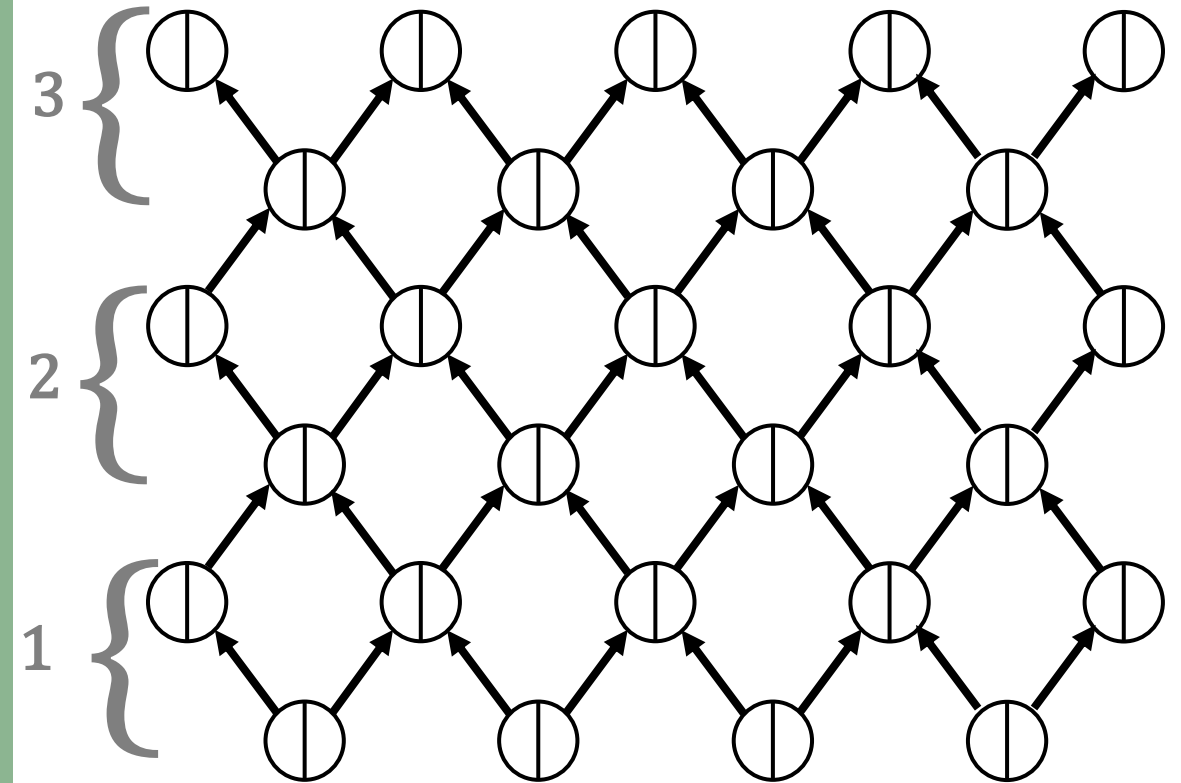
Causal structure

time



Causal graph rewriting

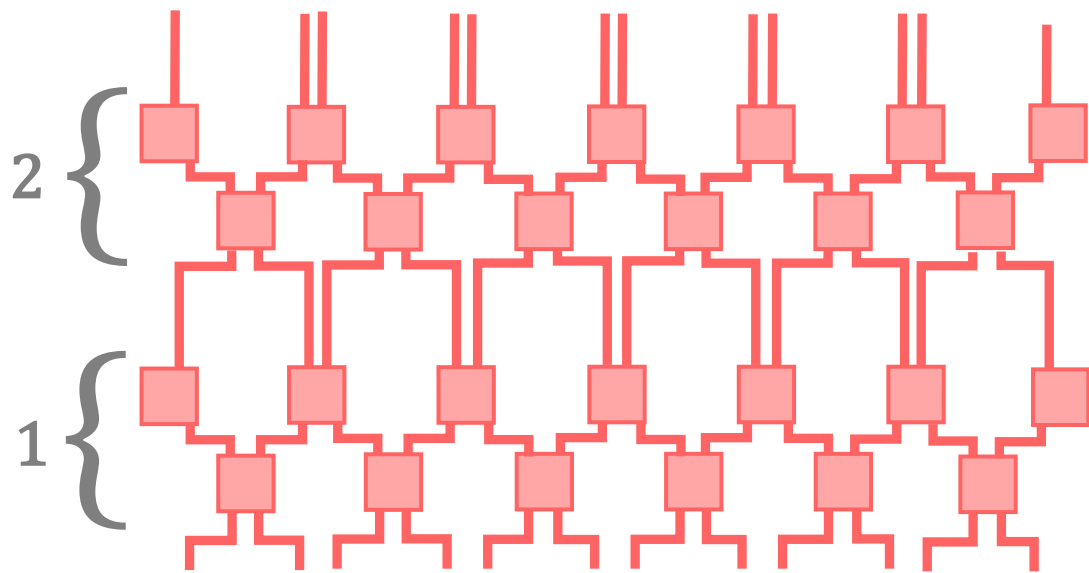
time



Dynamical systems and rewriting

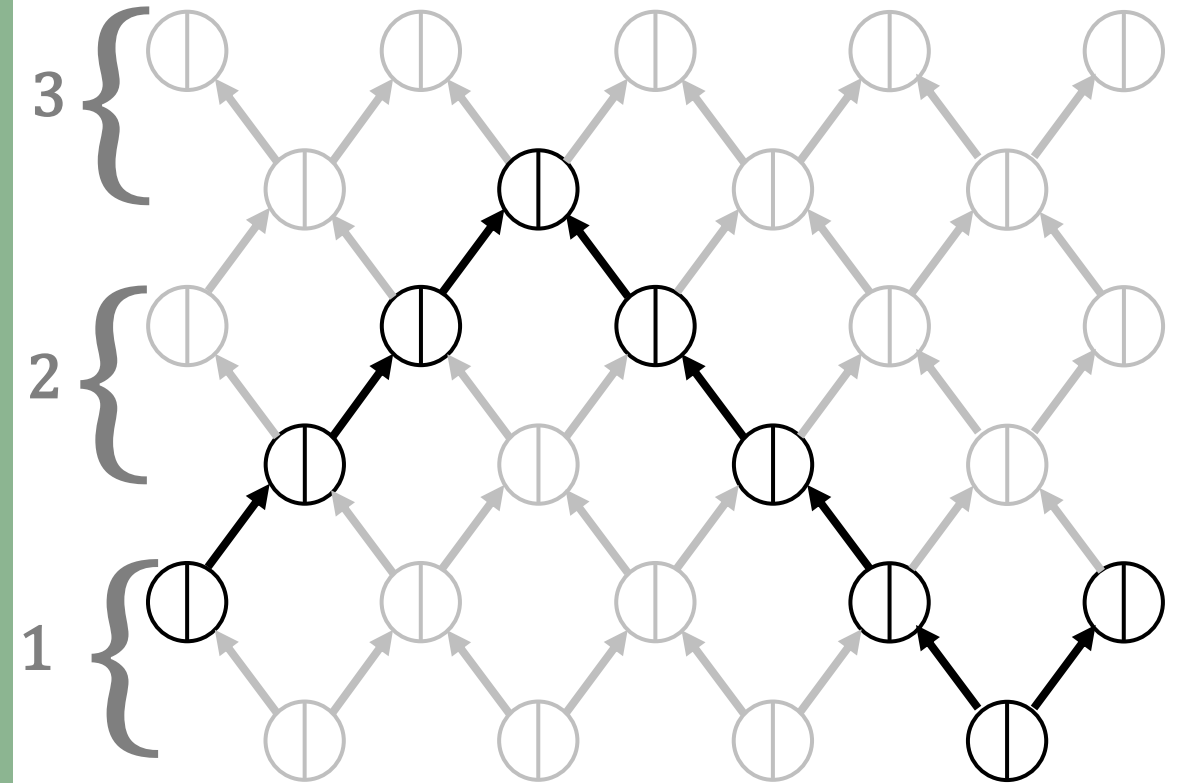
Causal structure

time



Causal graph rewriting

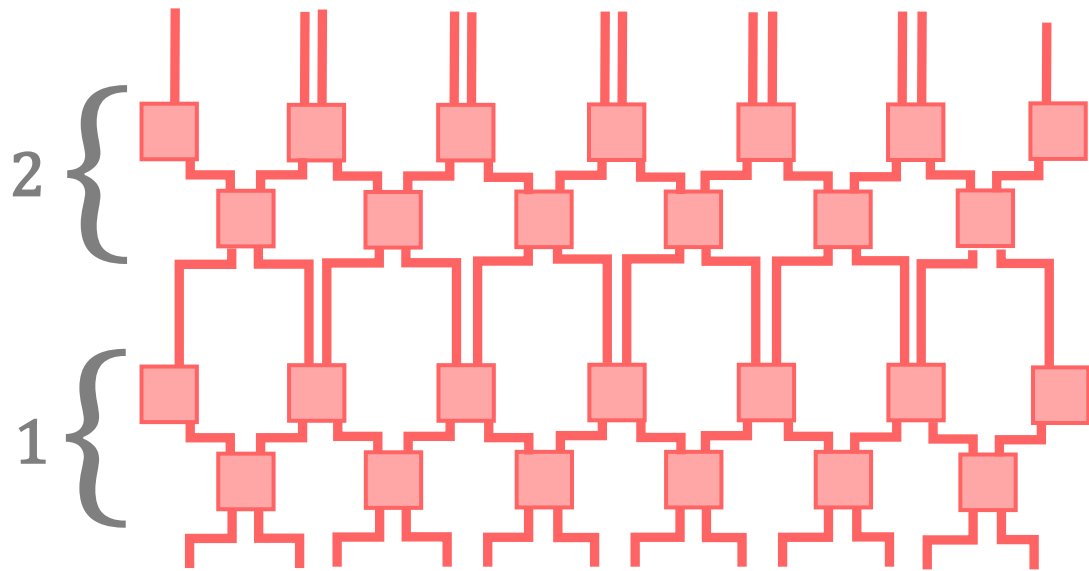
time



Dynamical systems and rewriting

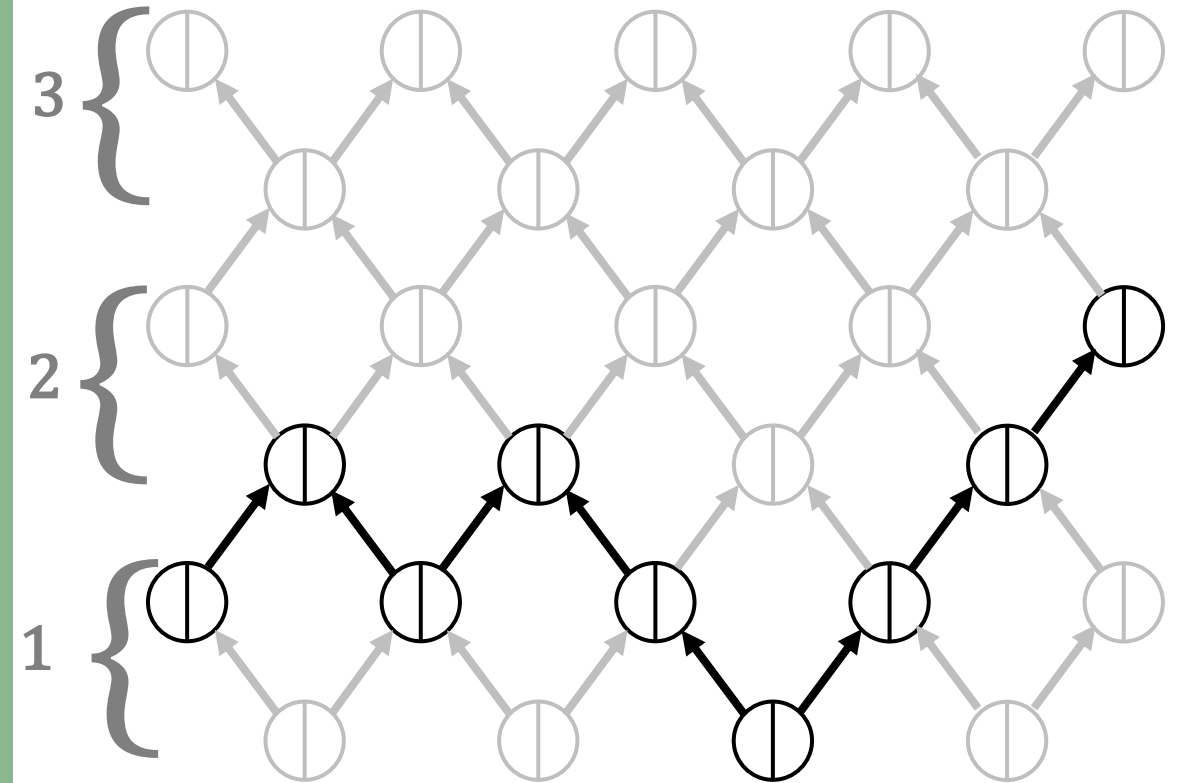
Causal structure

time



Causal graph rewriting

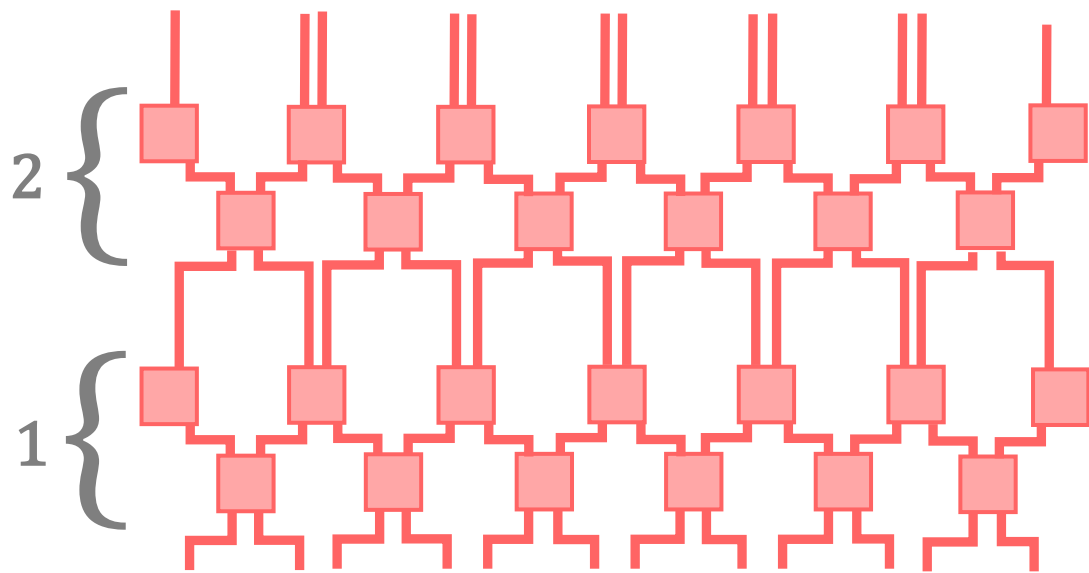
time



Dynamical systems and rewriting

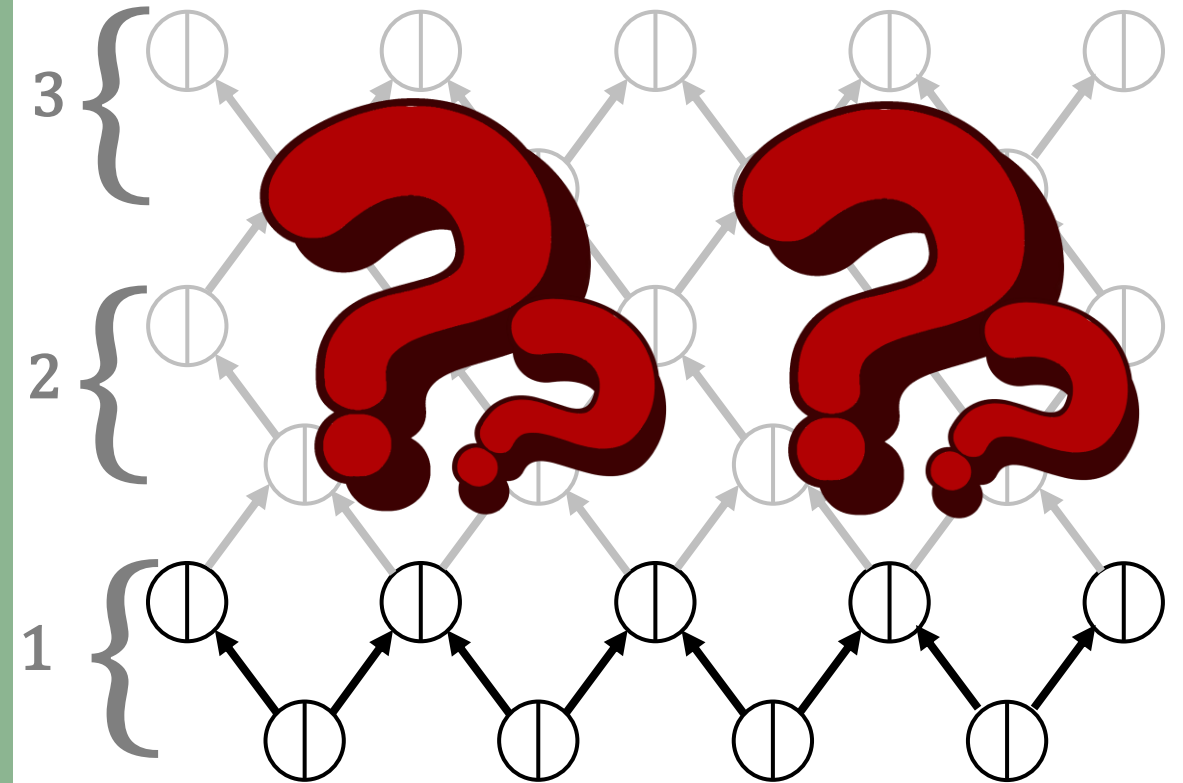
Causal structure

time

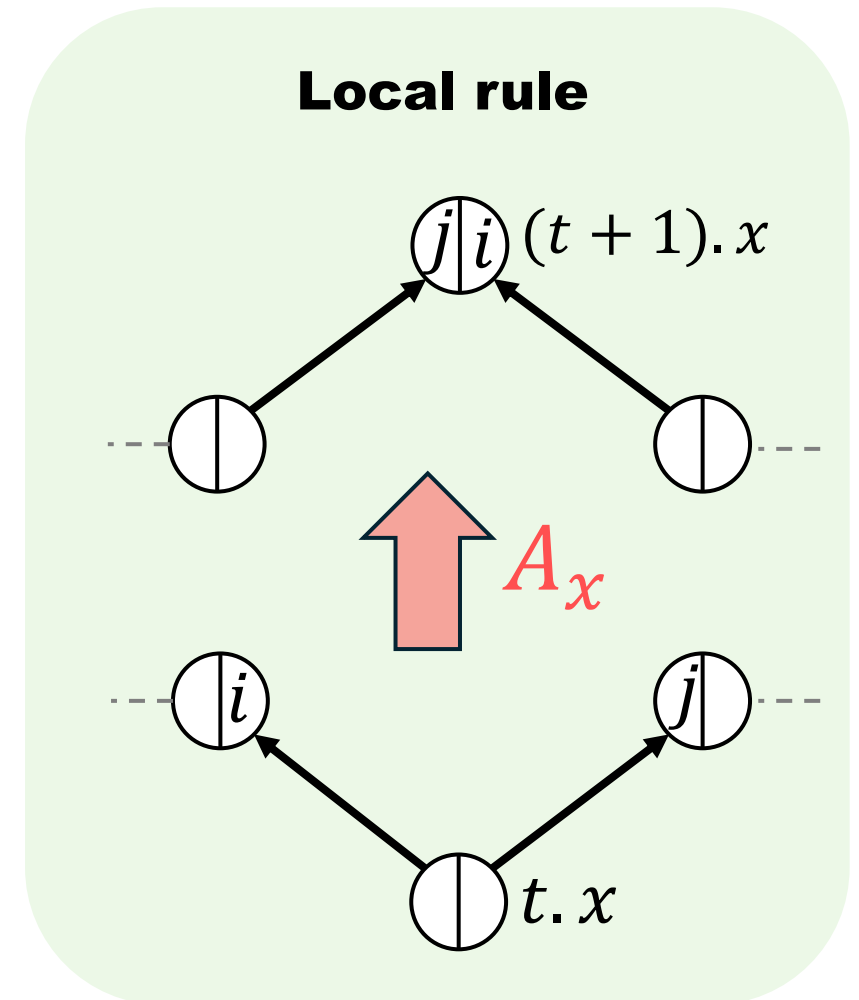
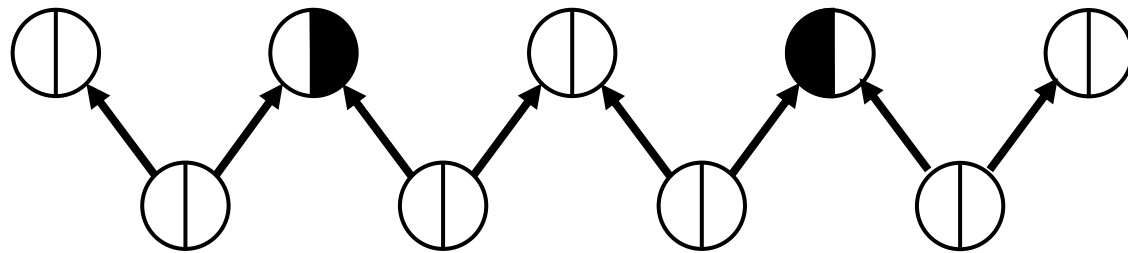


Causal graph rewriting

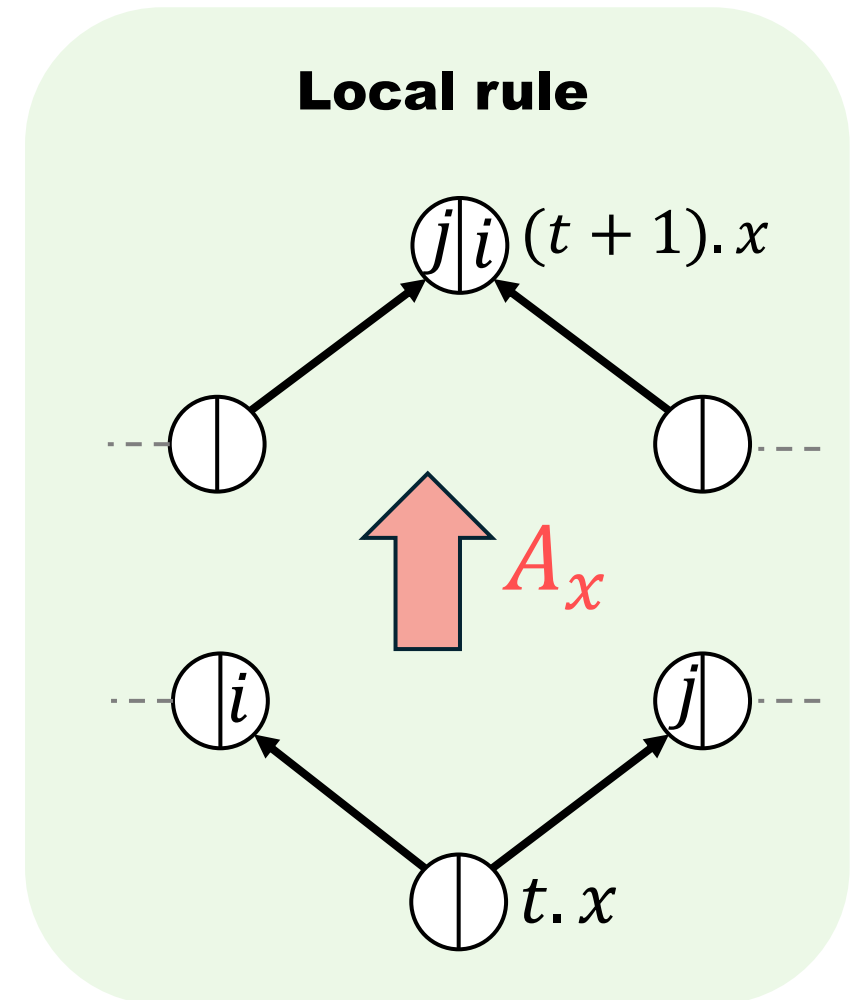
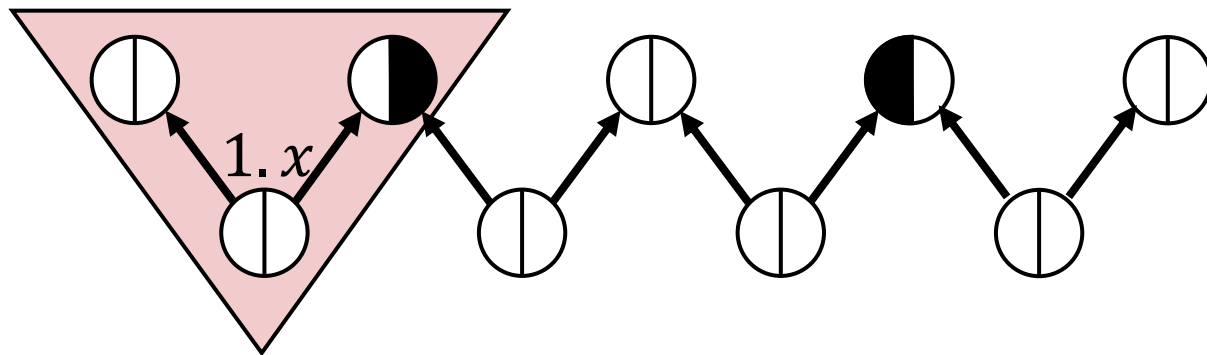
time



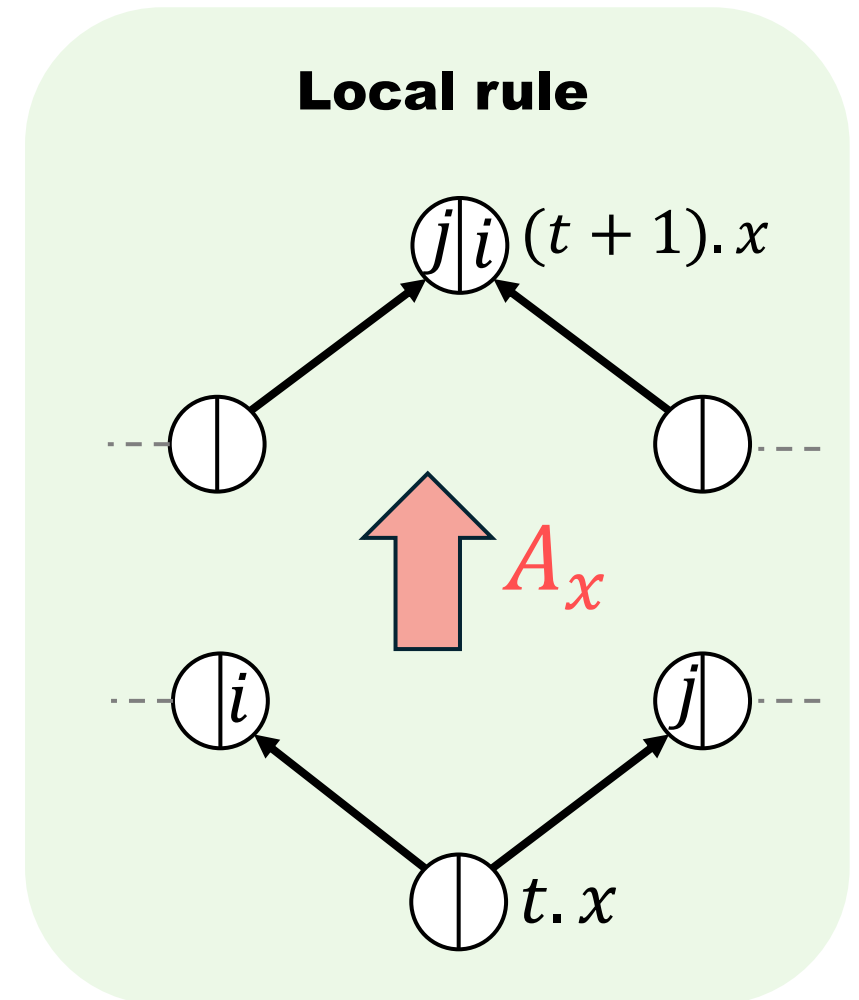
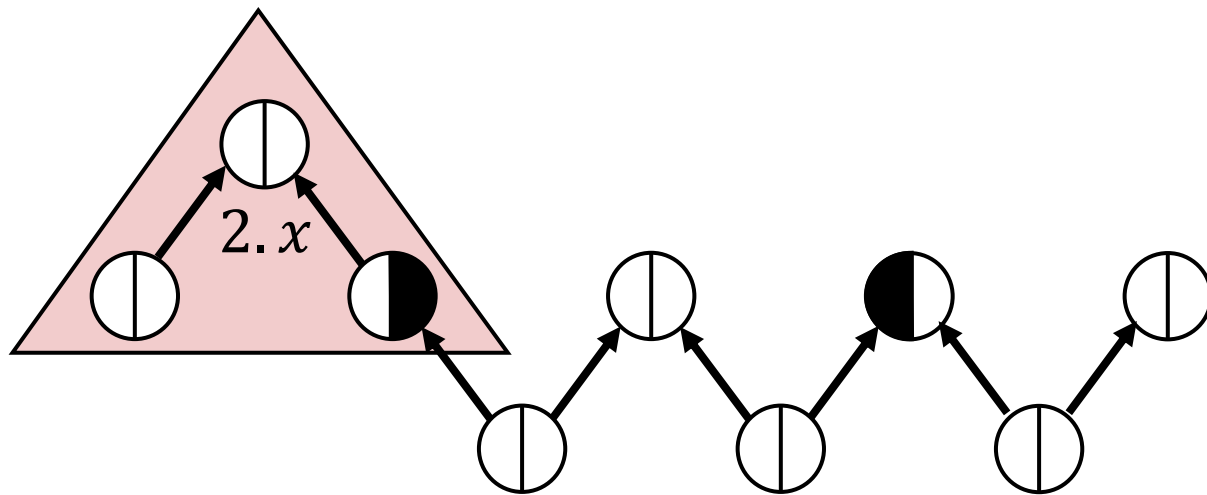
Particle system example



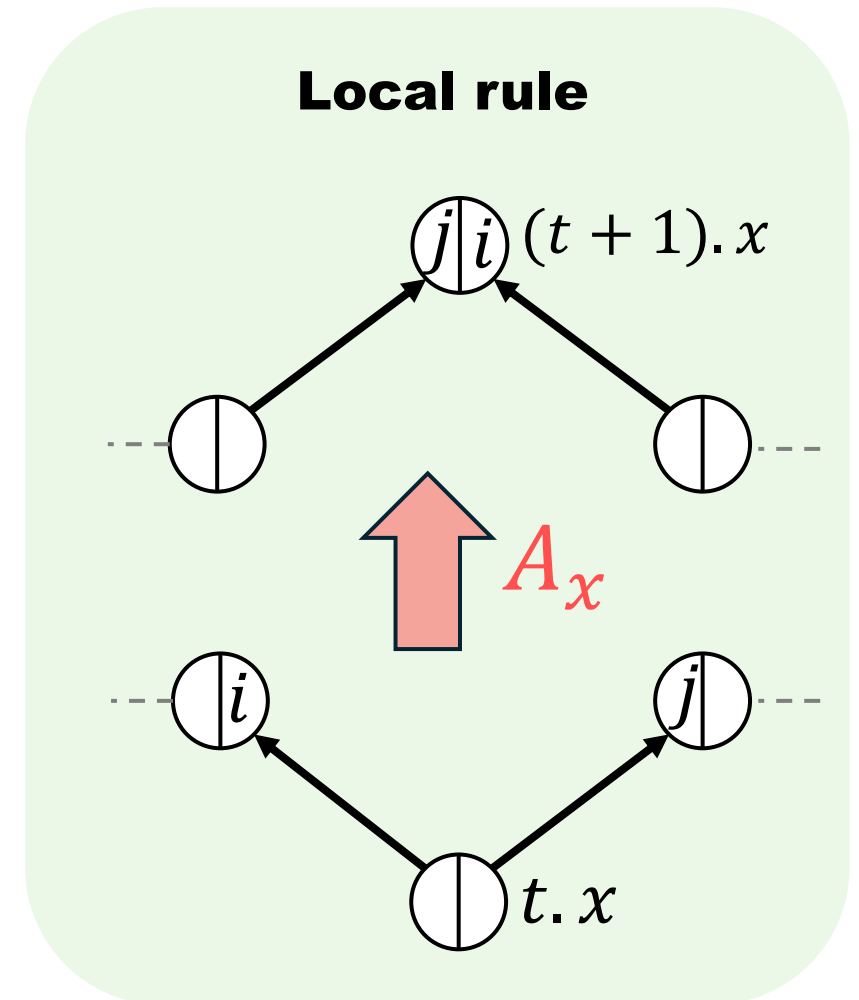
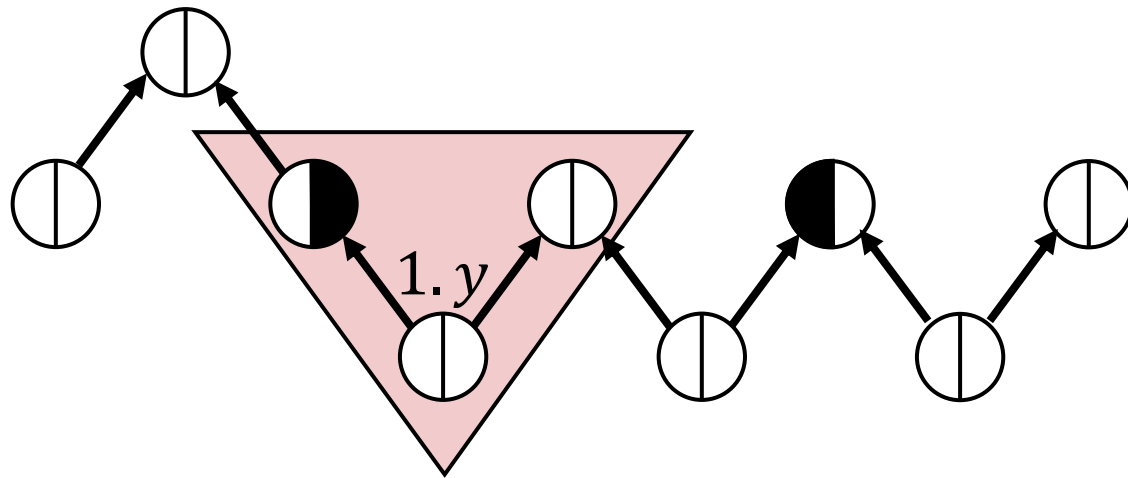
Particle system example



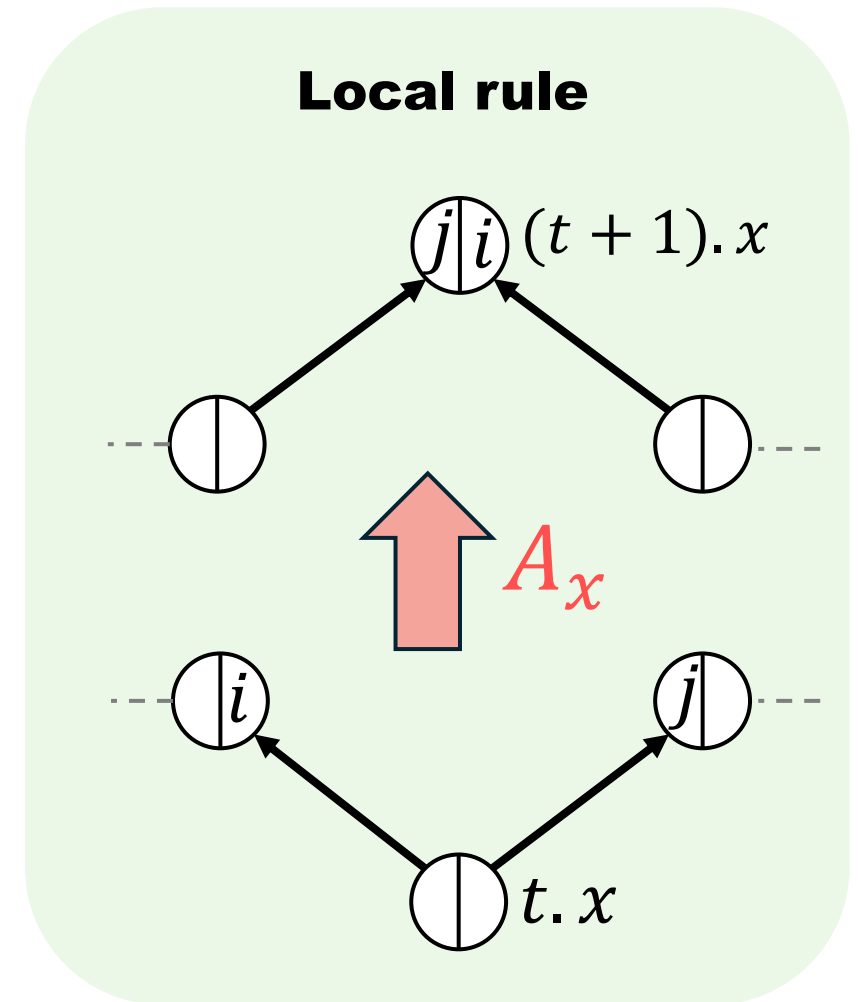
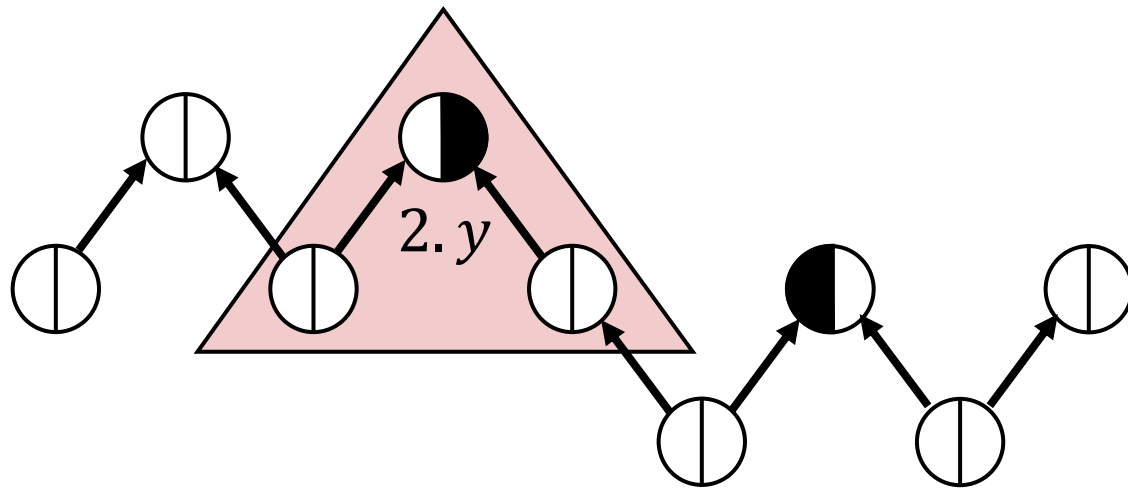
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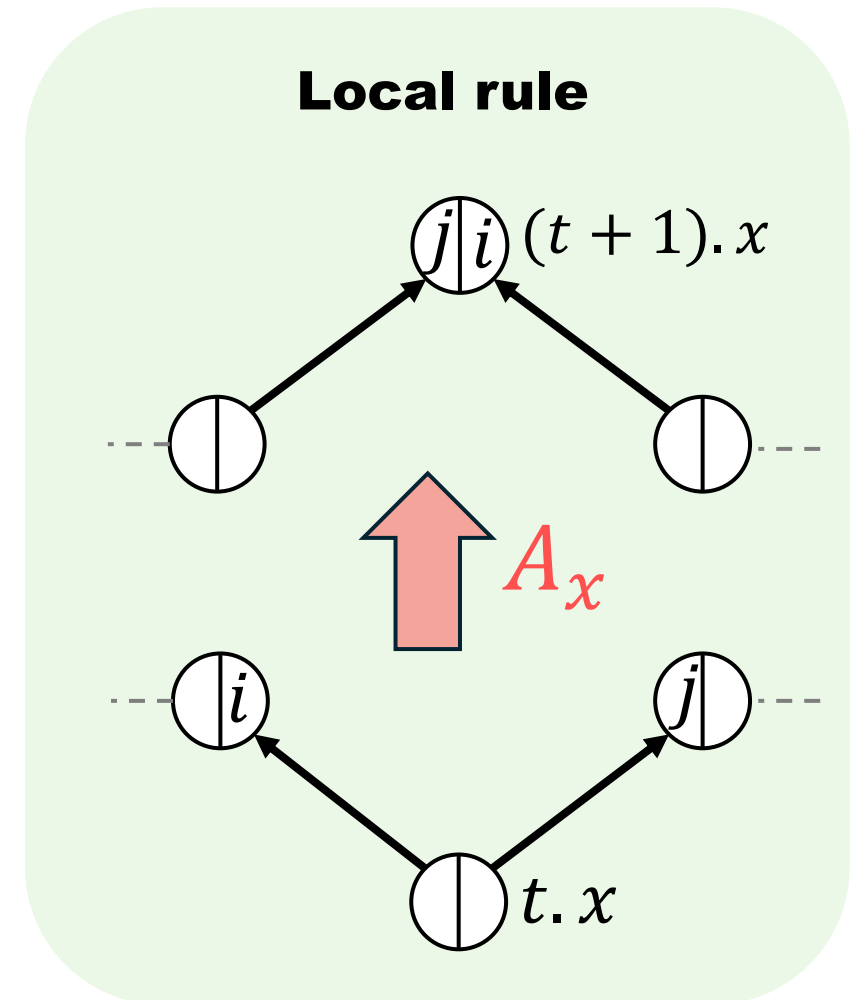
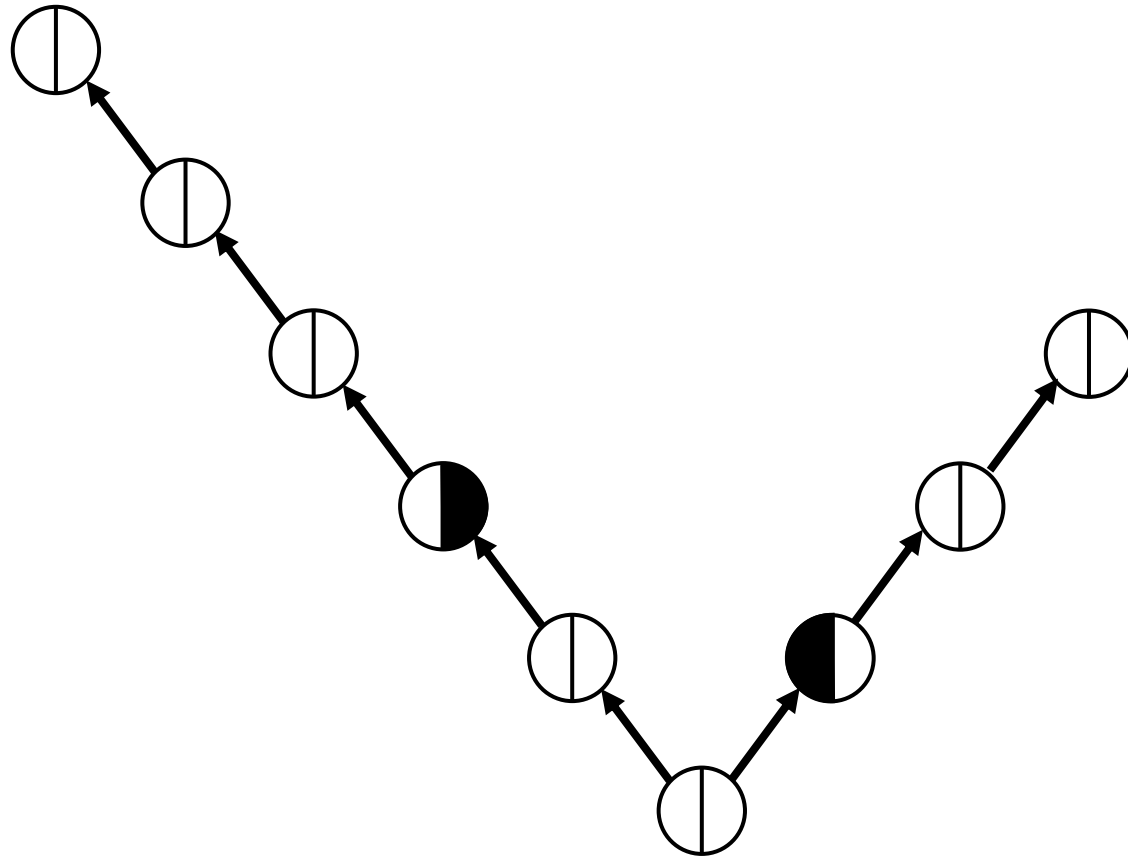
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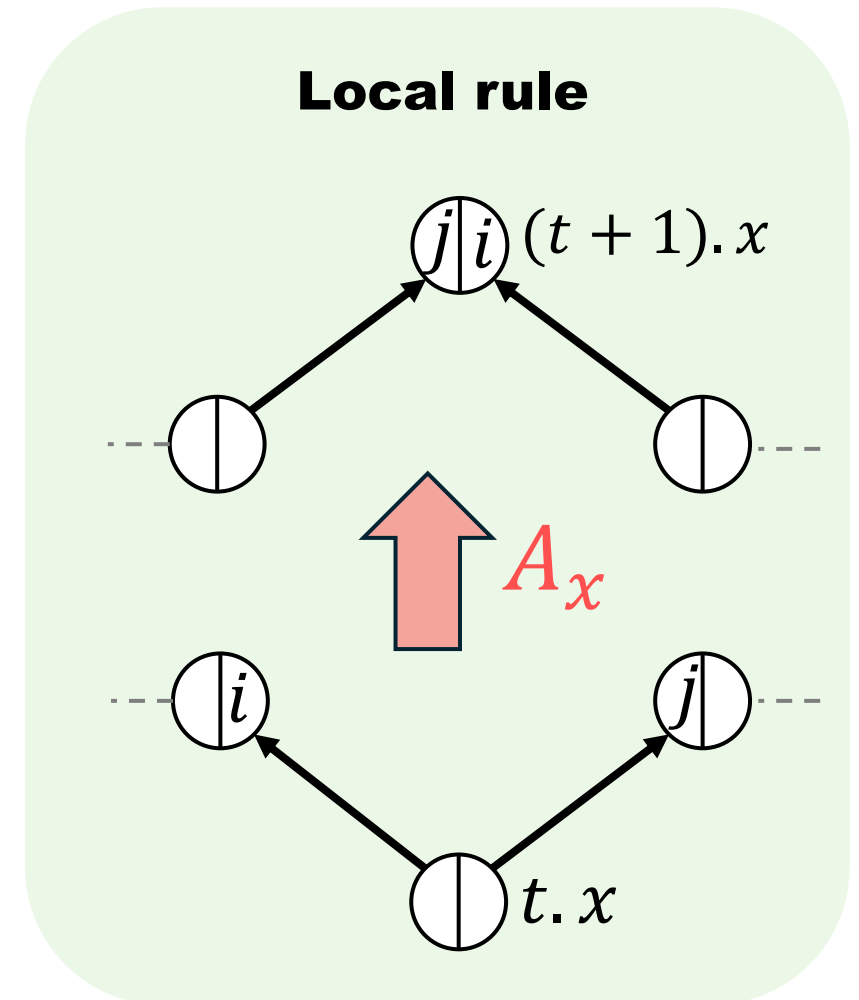
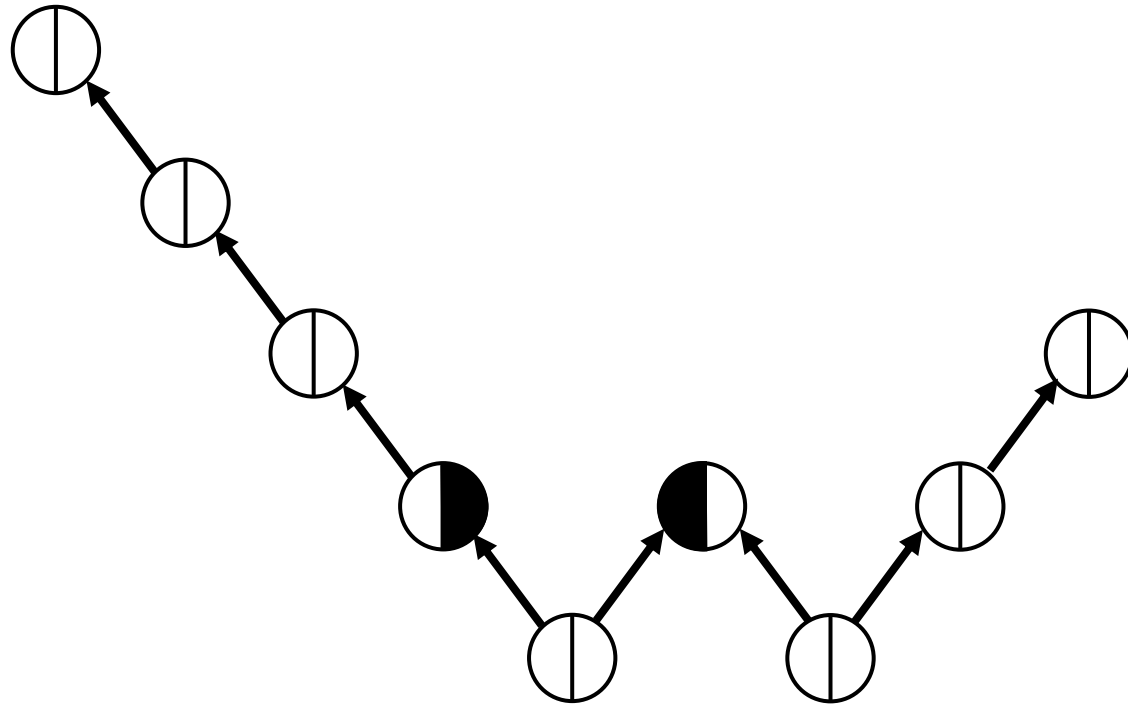
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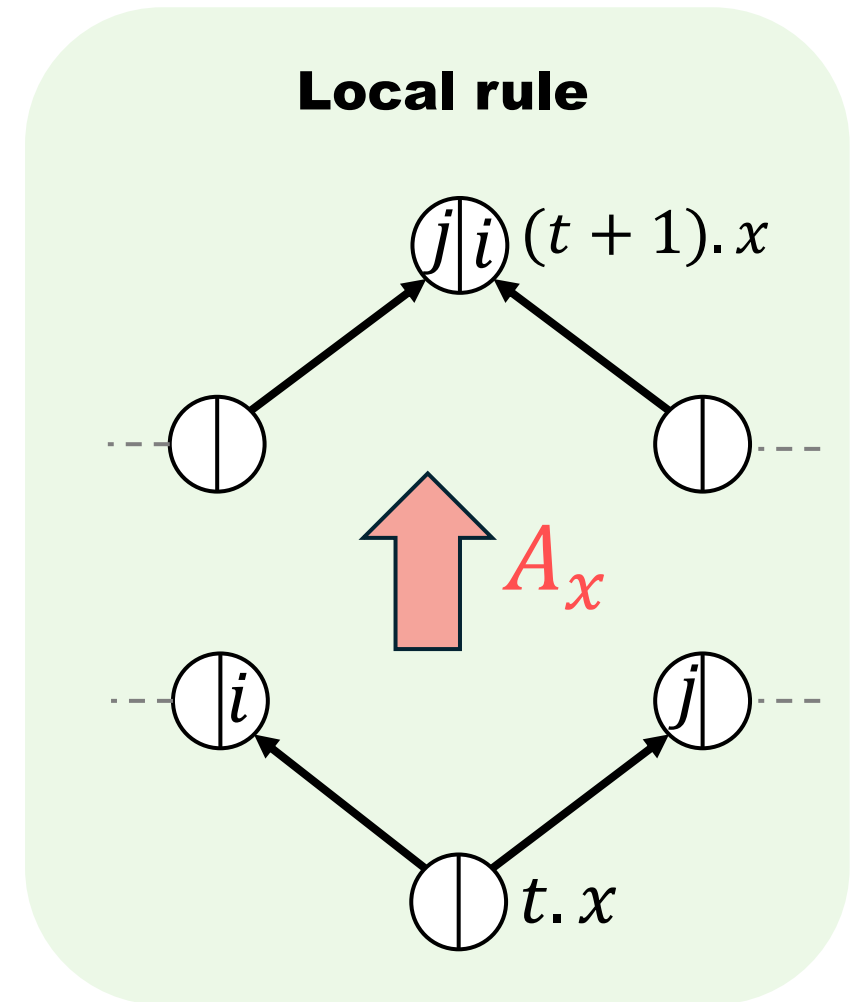
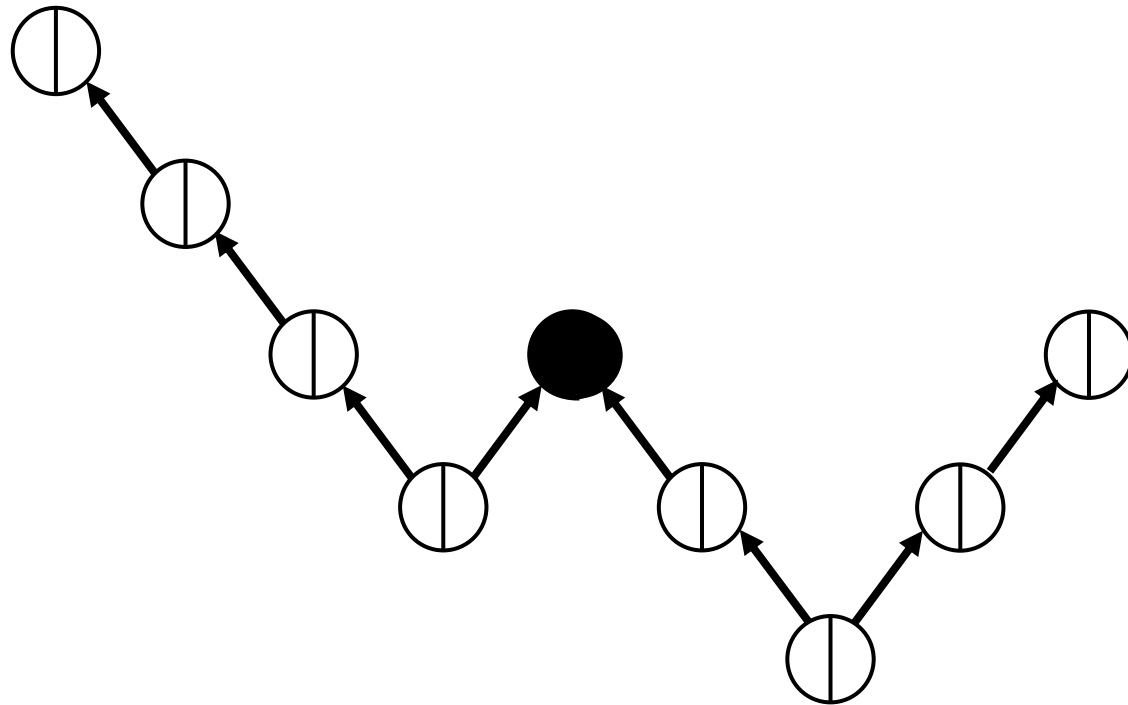
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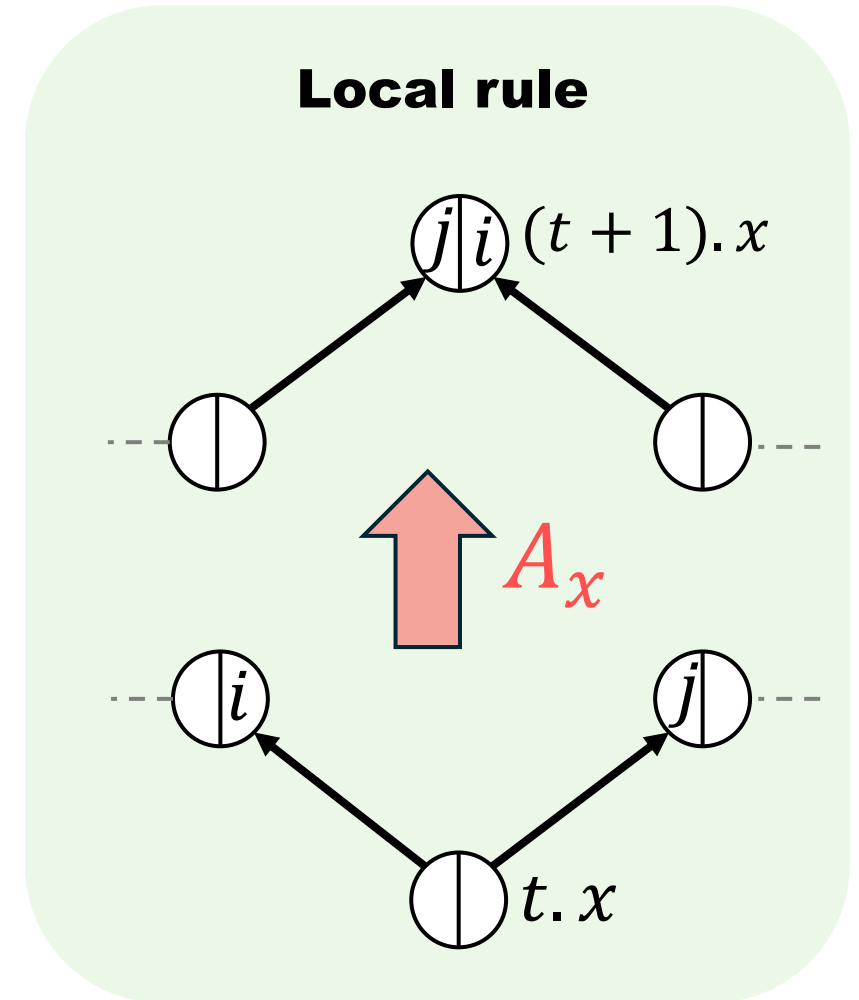
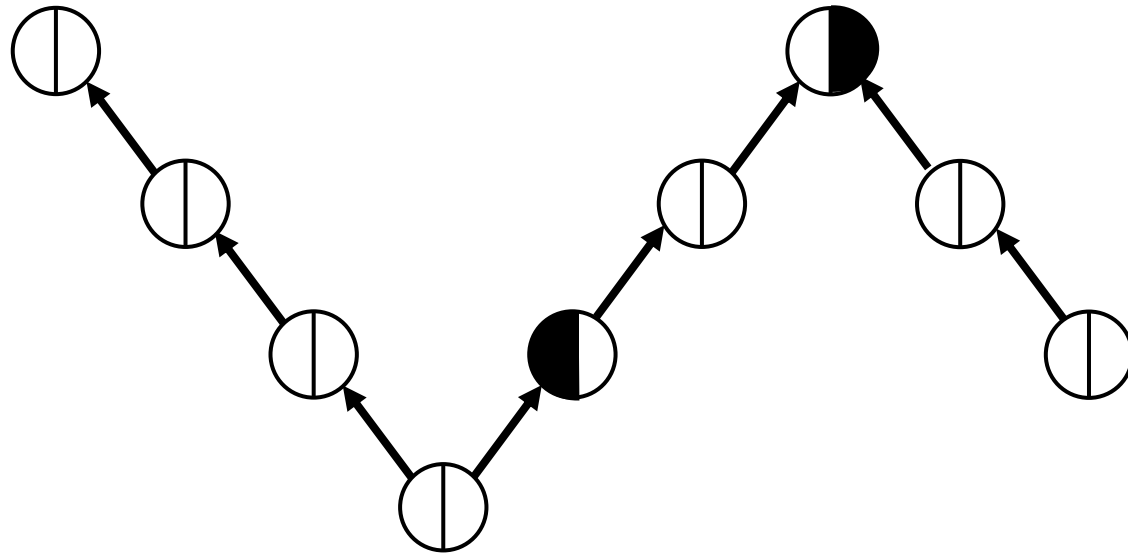
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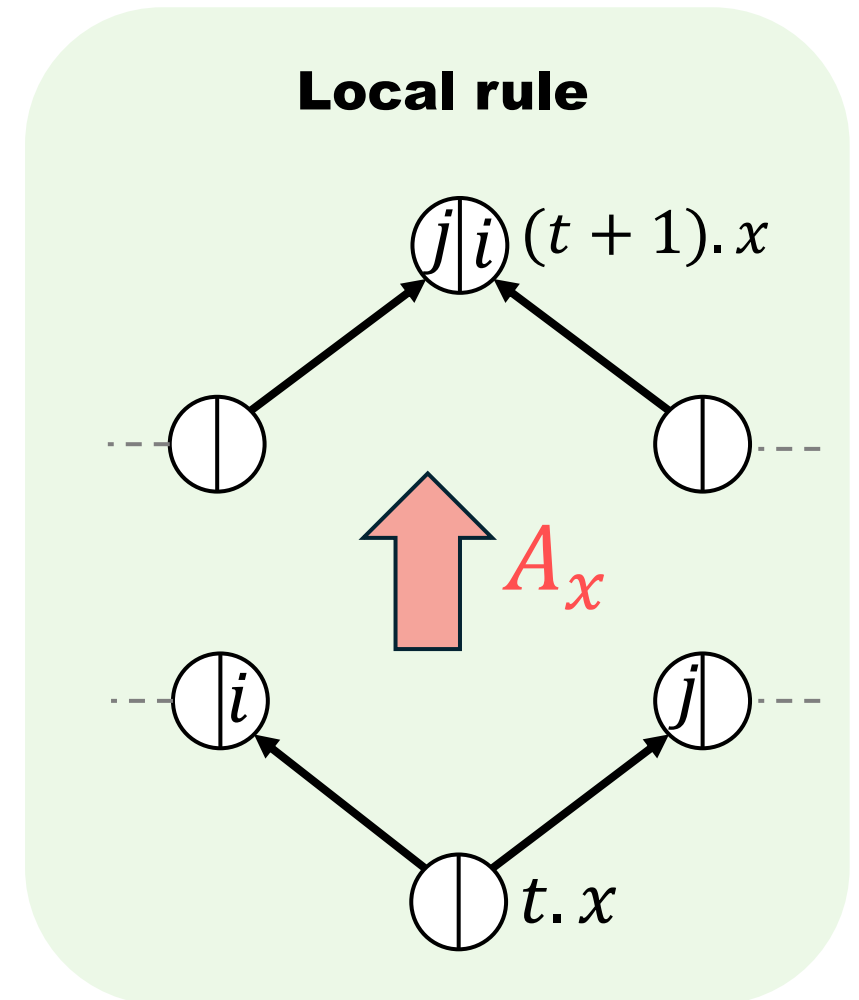
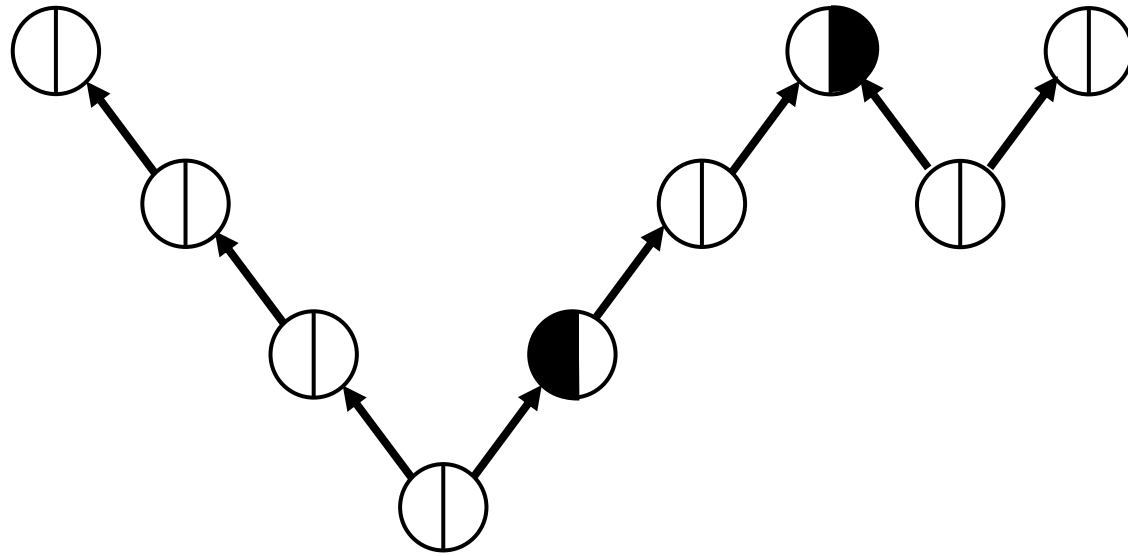
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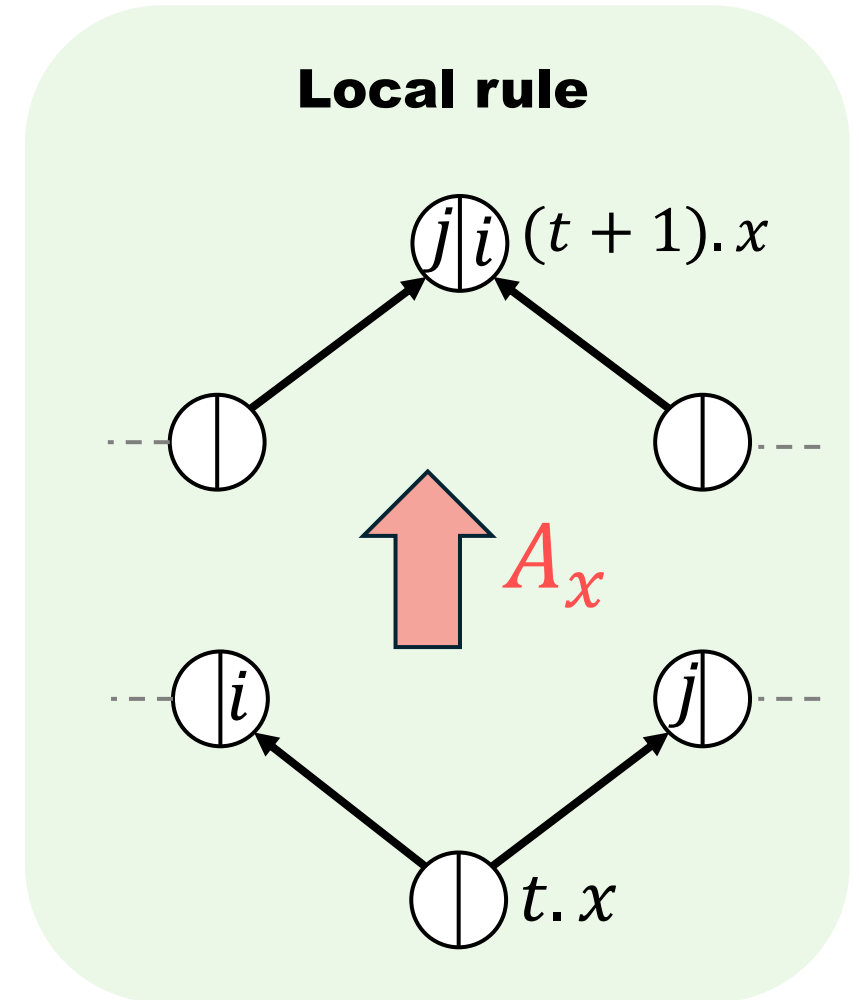
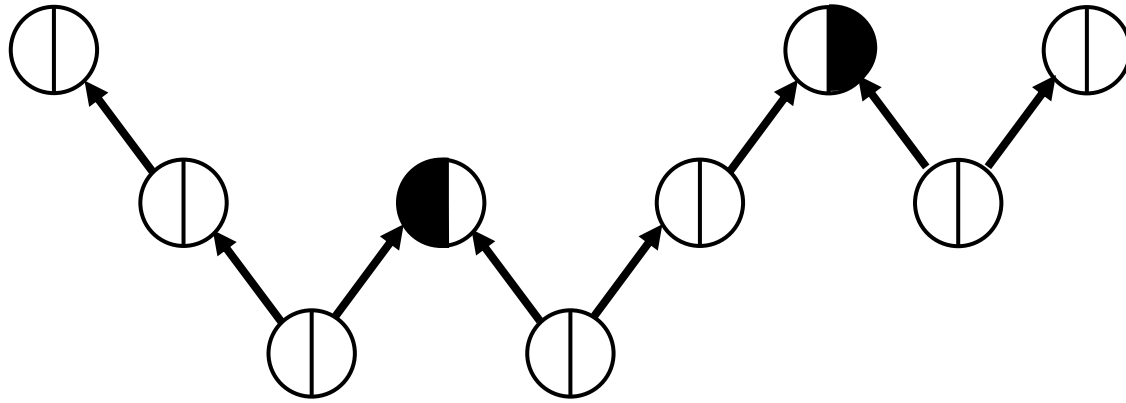
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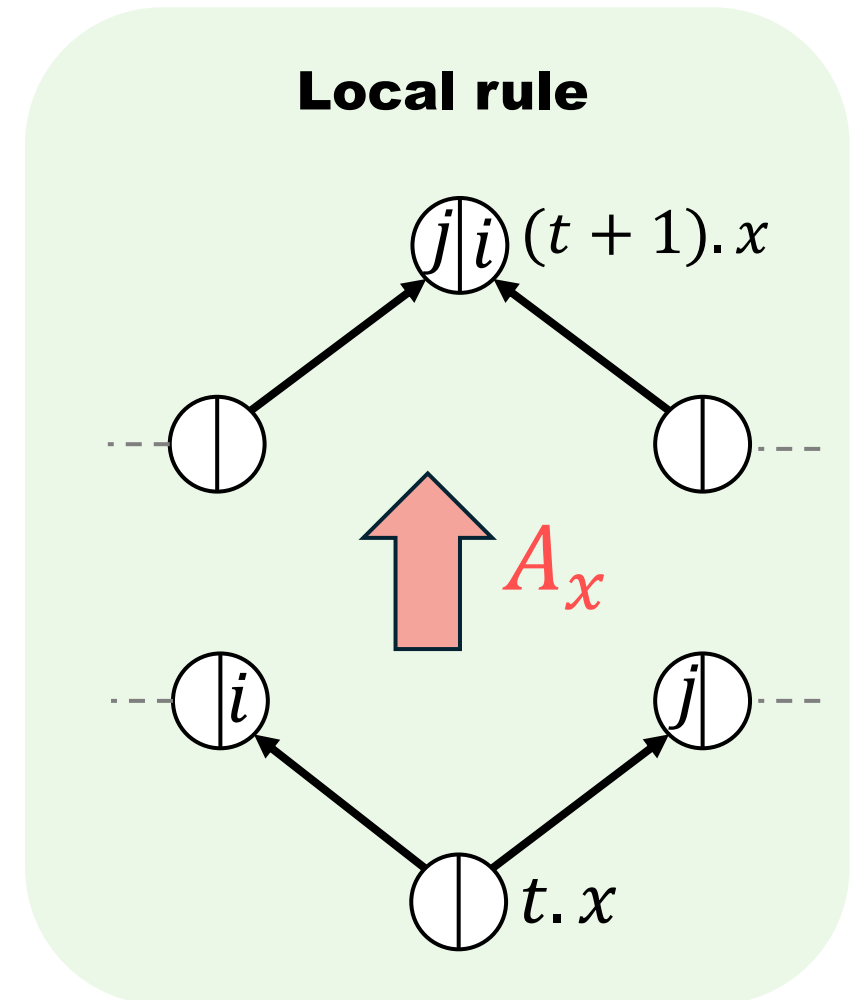
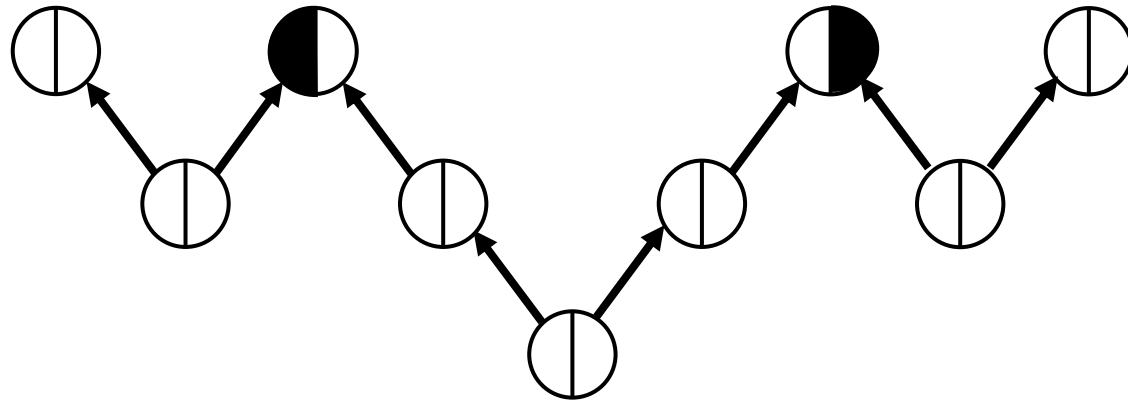
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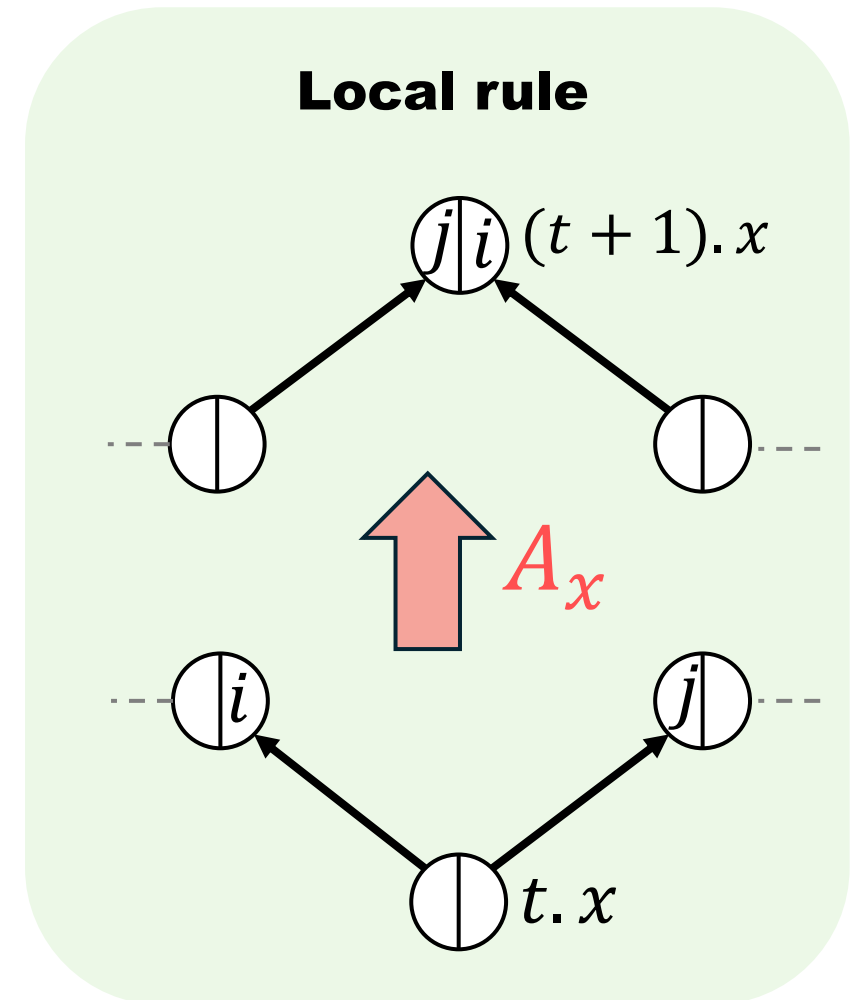
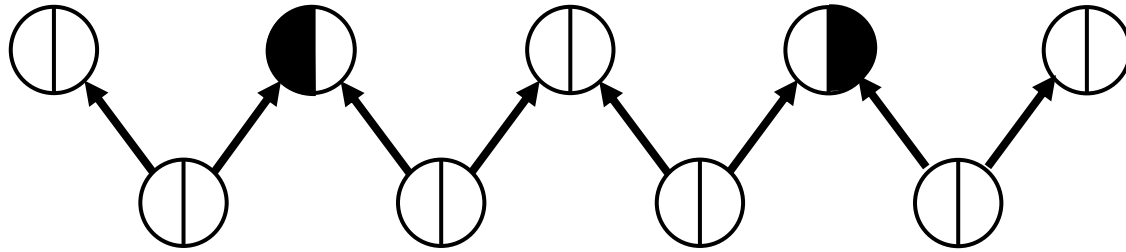
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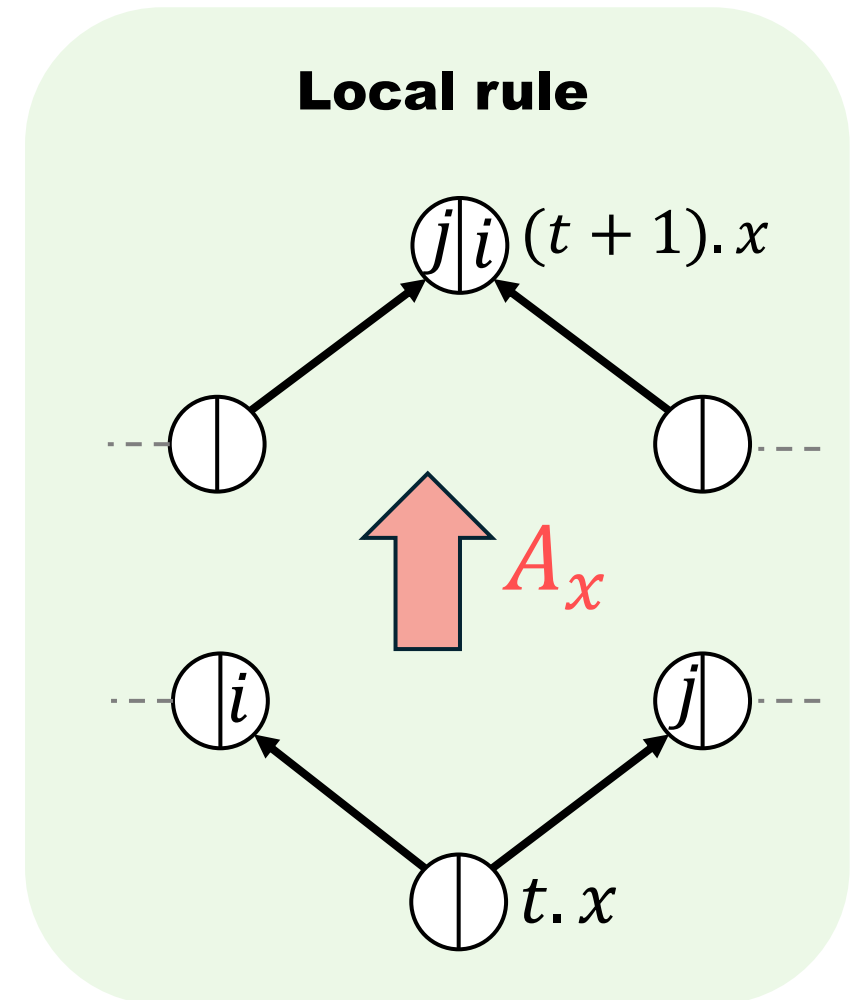
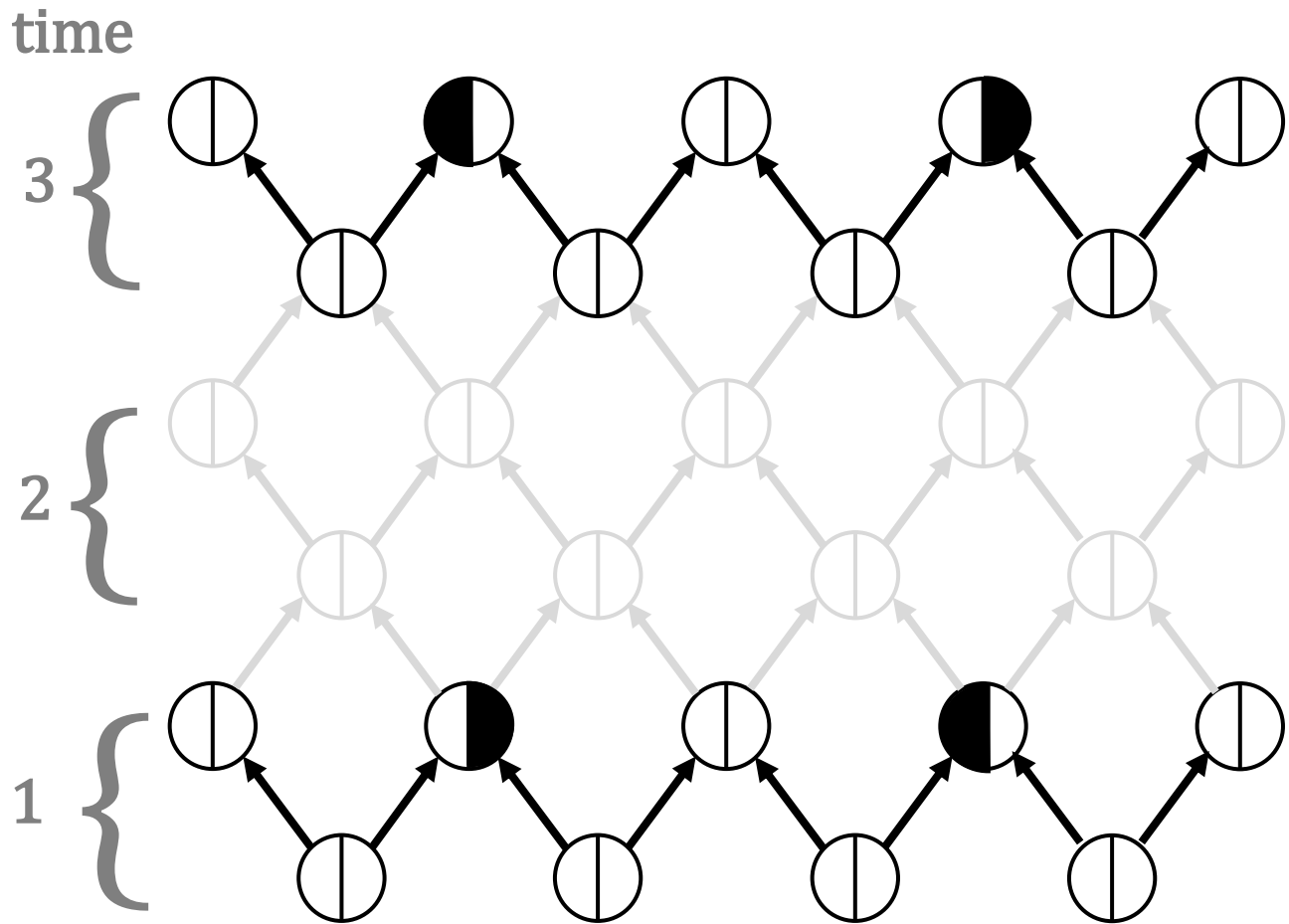
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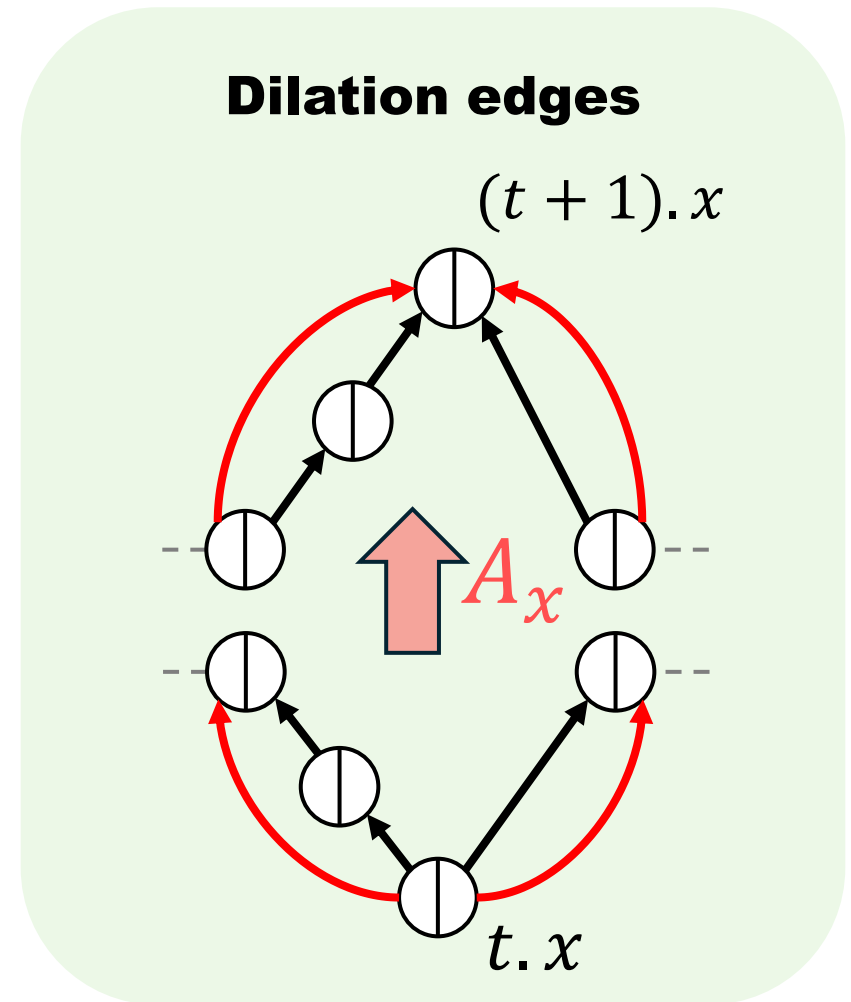
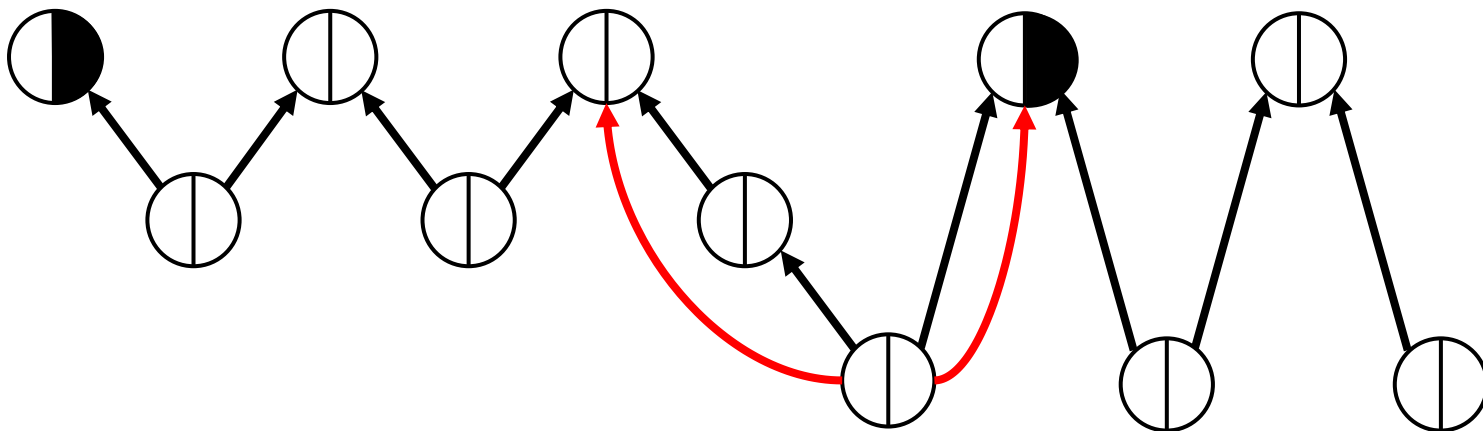
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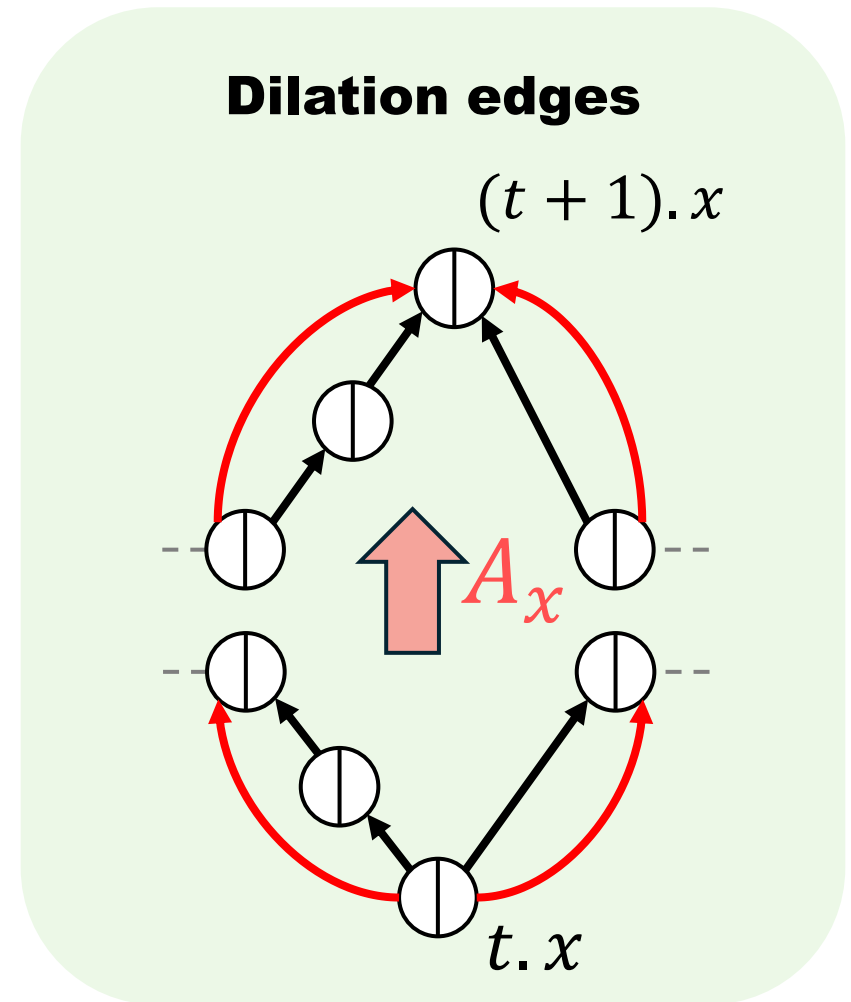
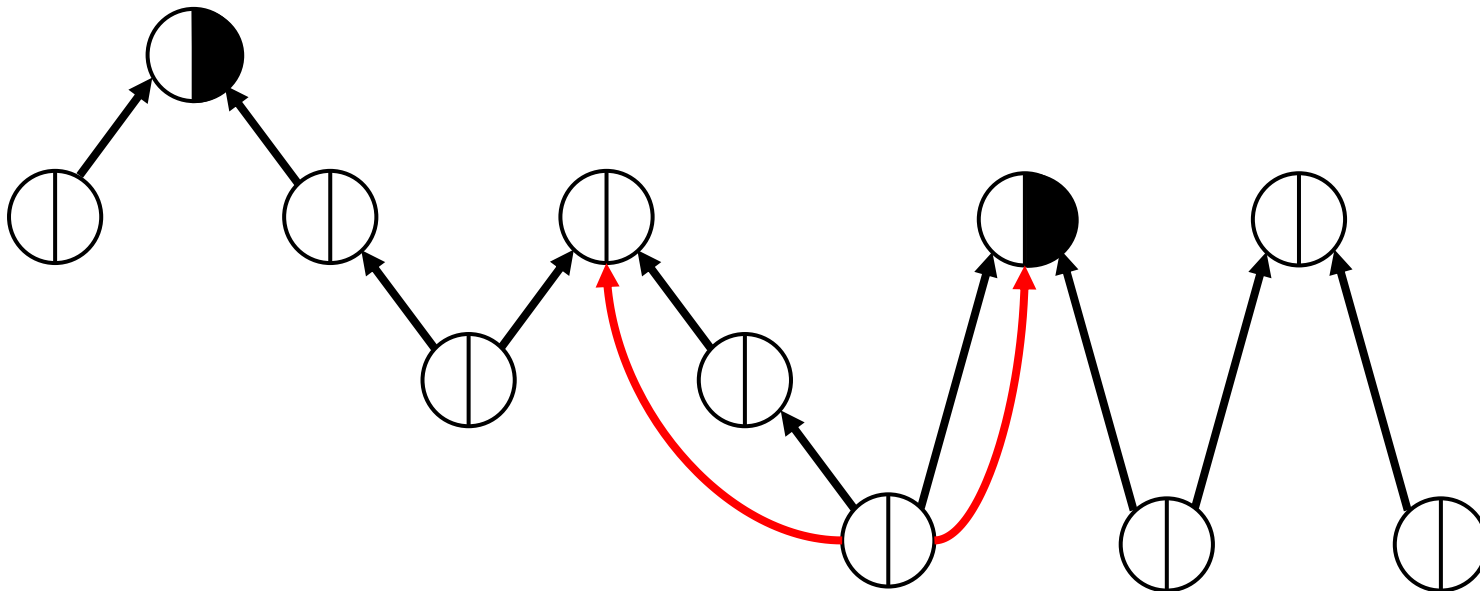
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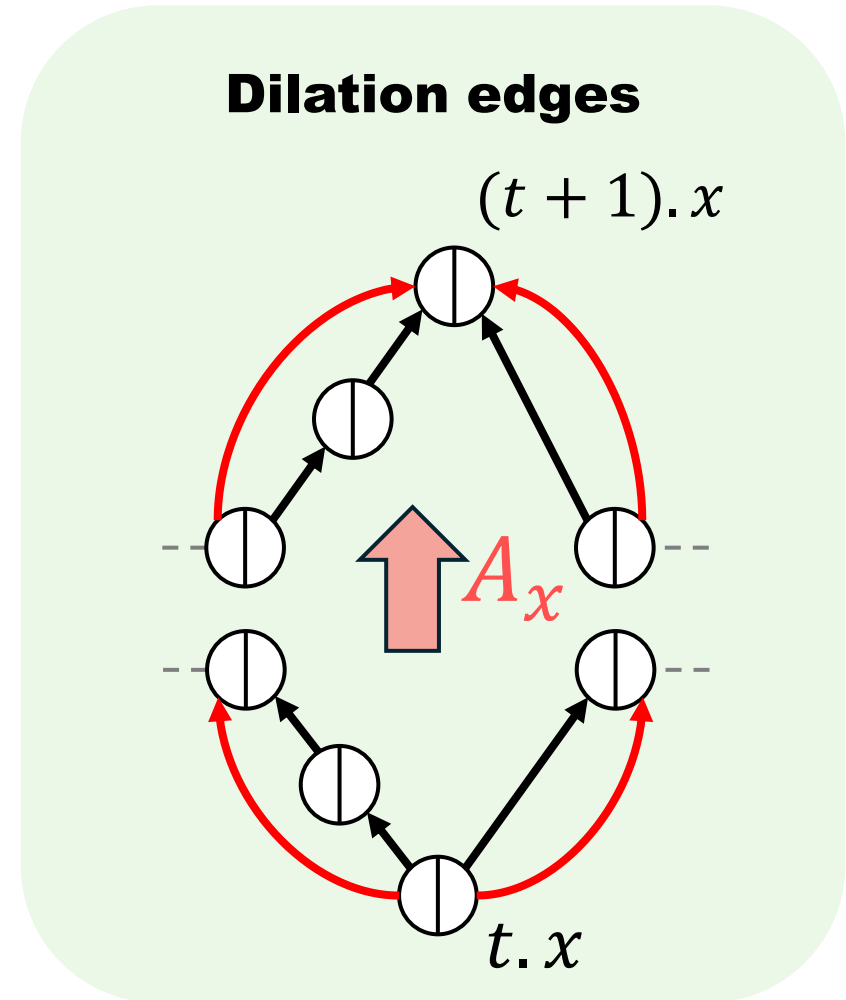
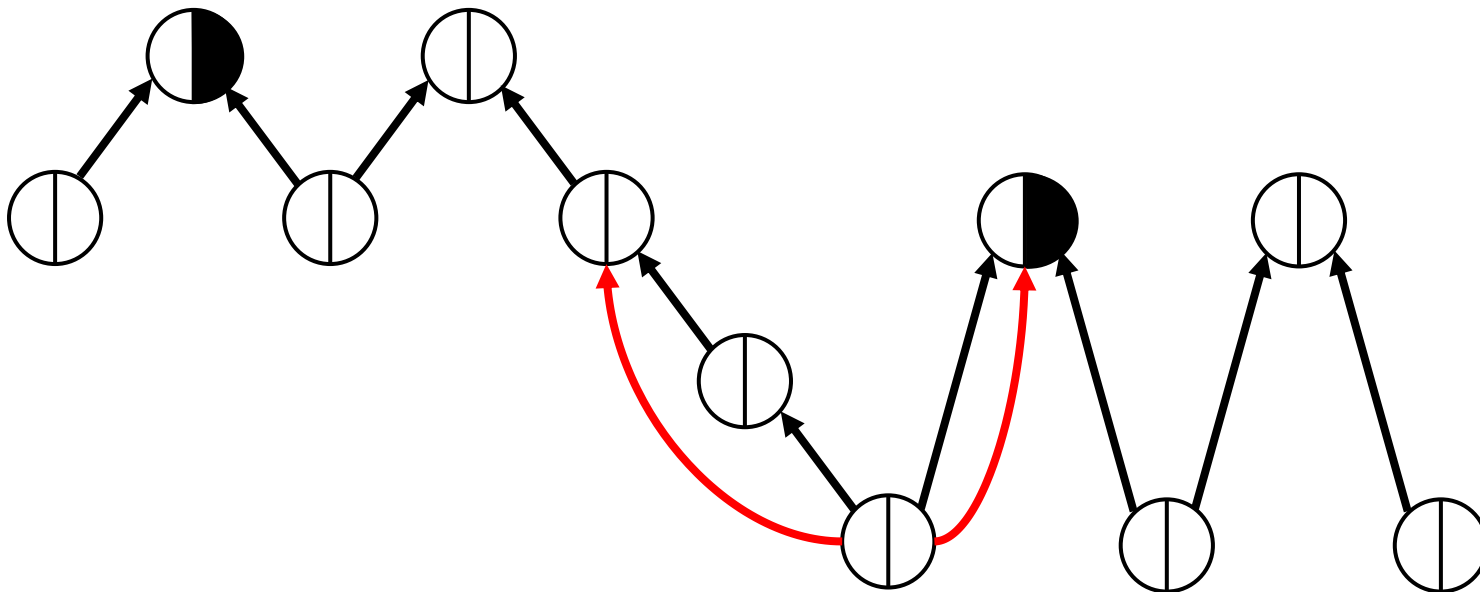
Time dilation example



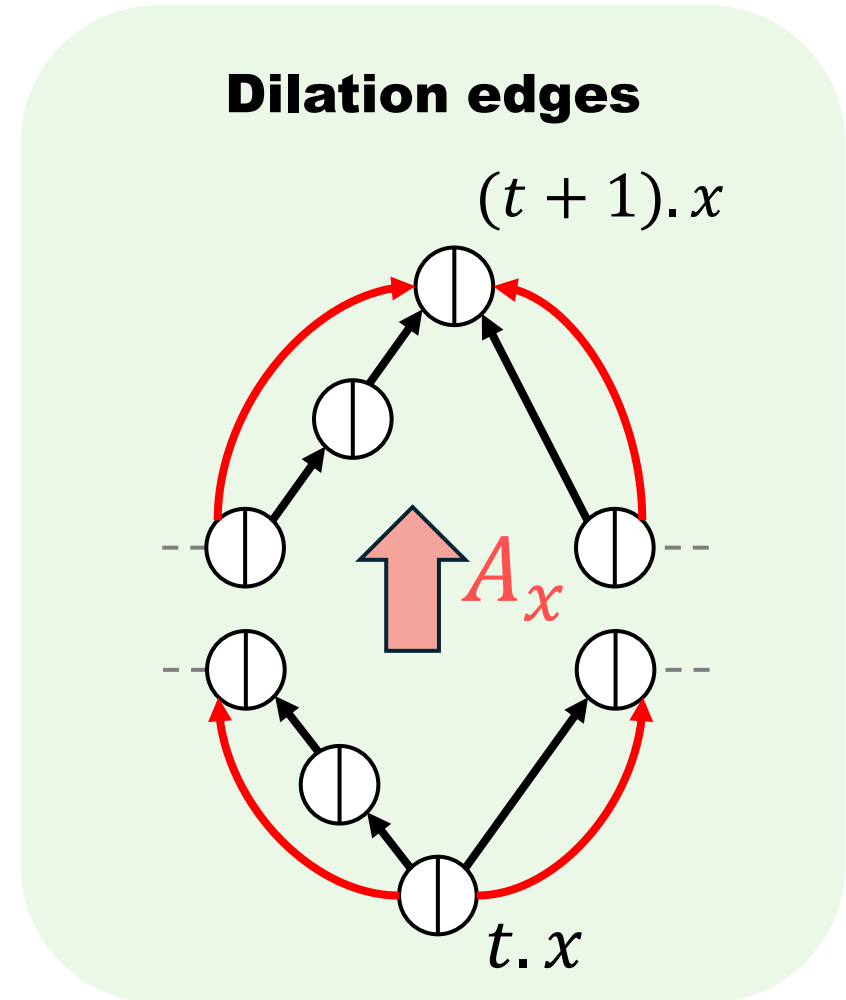
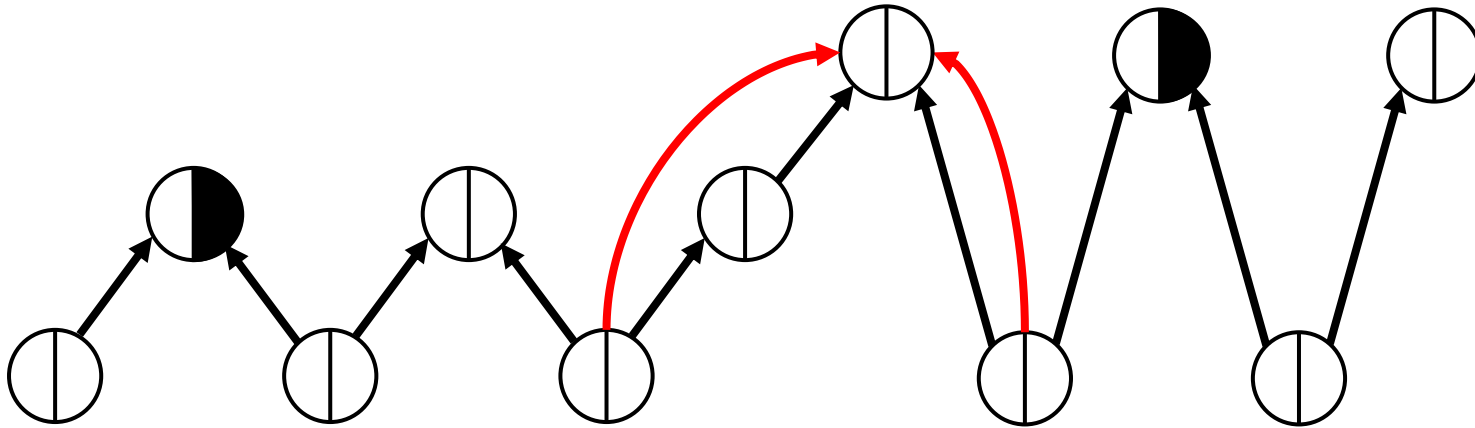
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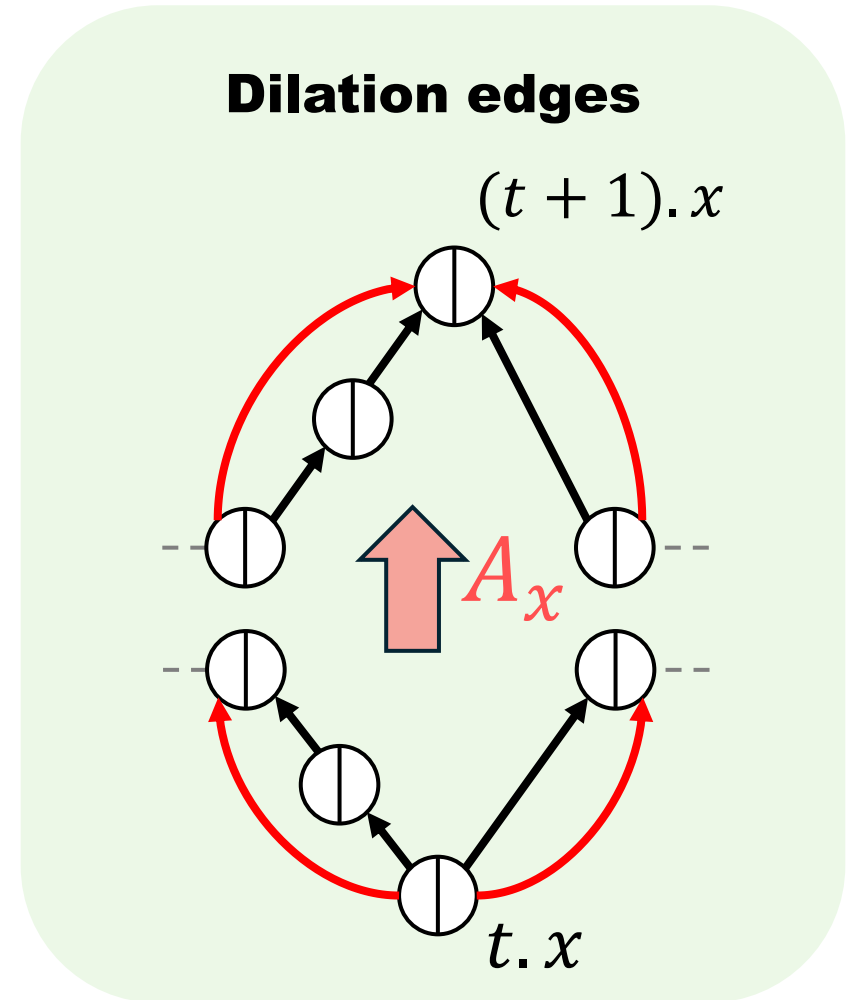
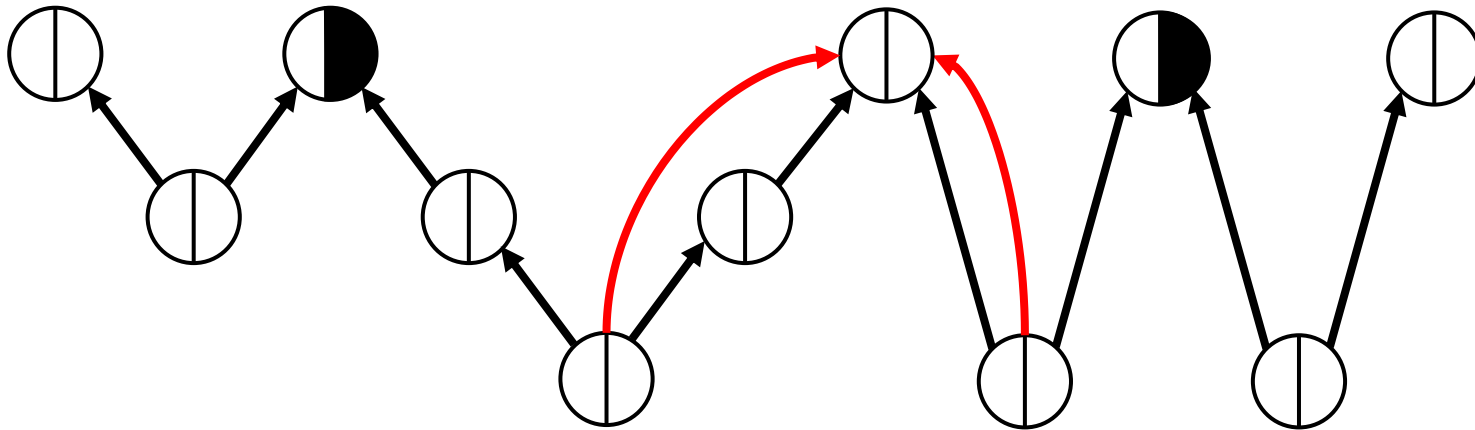
Time dilation example



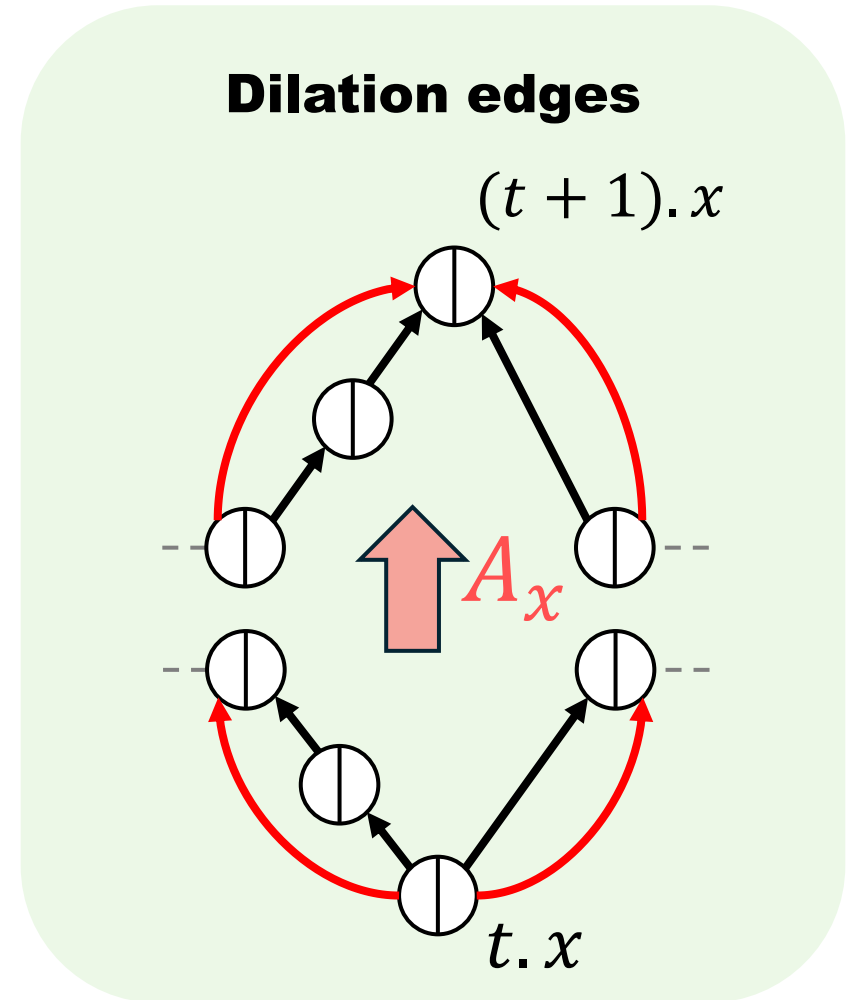
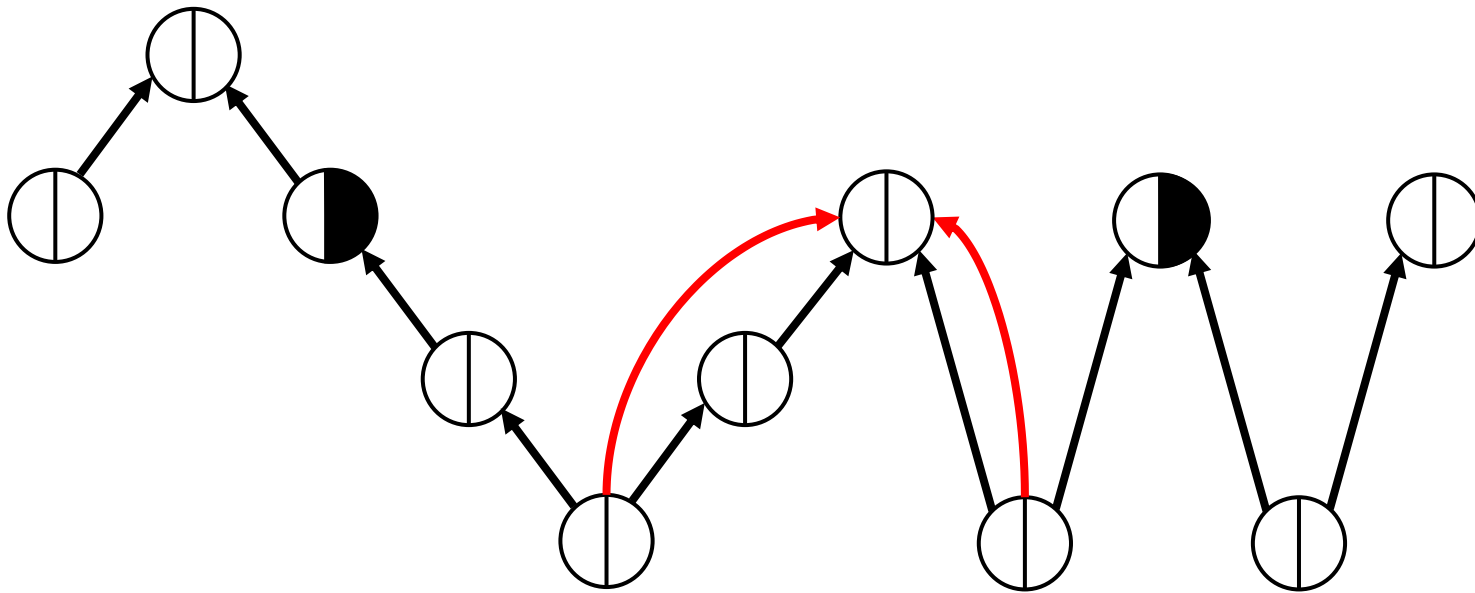
Time dilation example



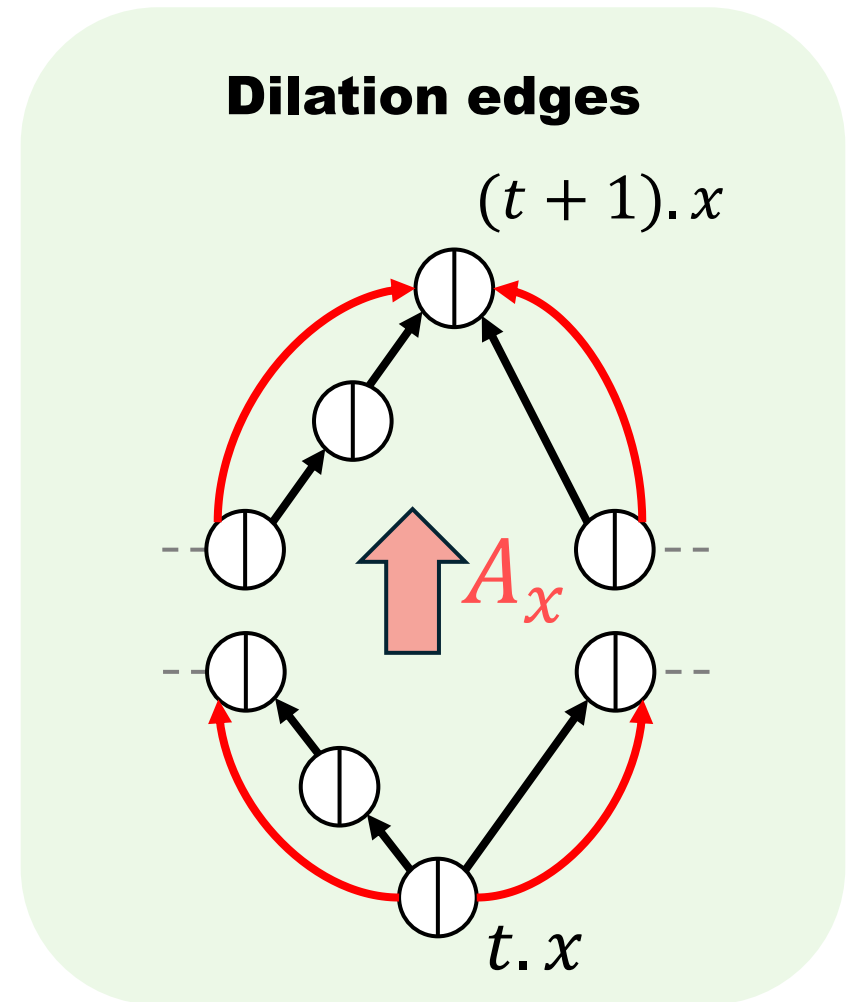
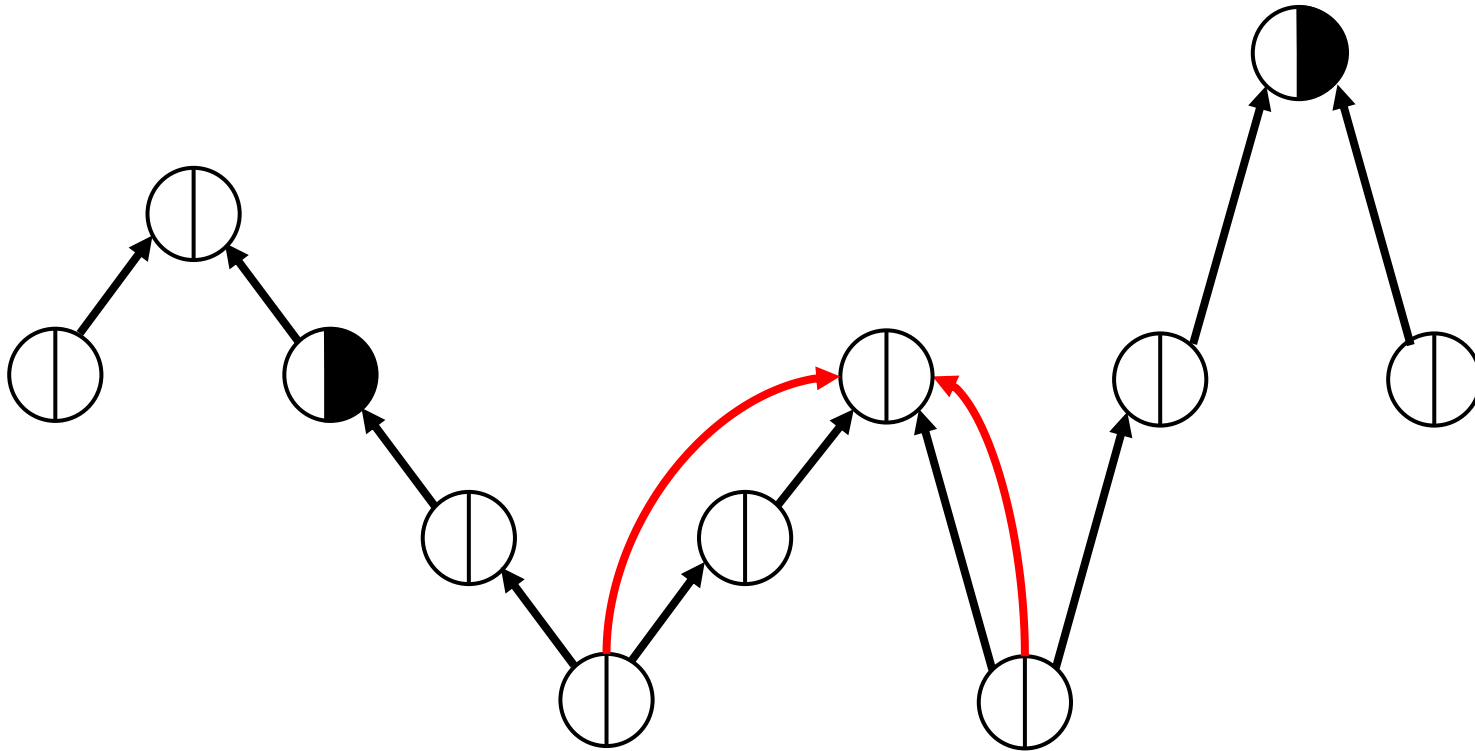
Time dilation example



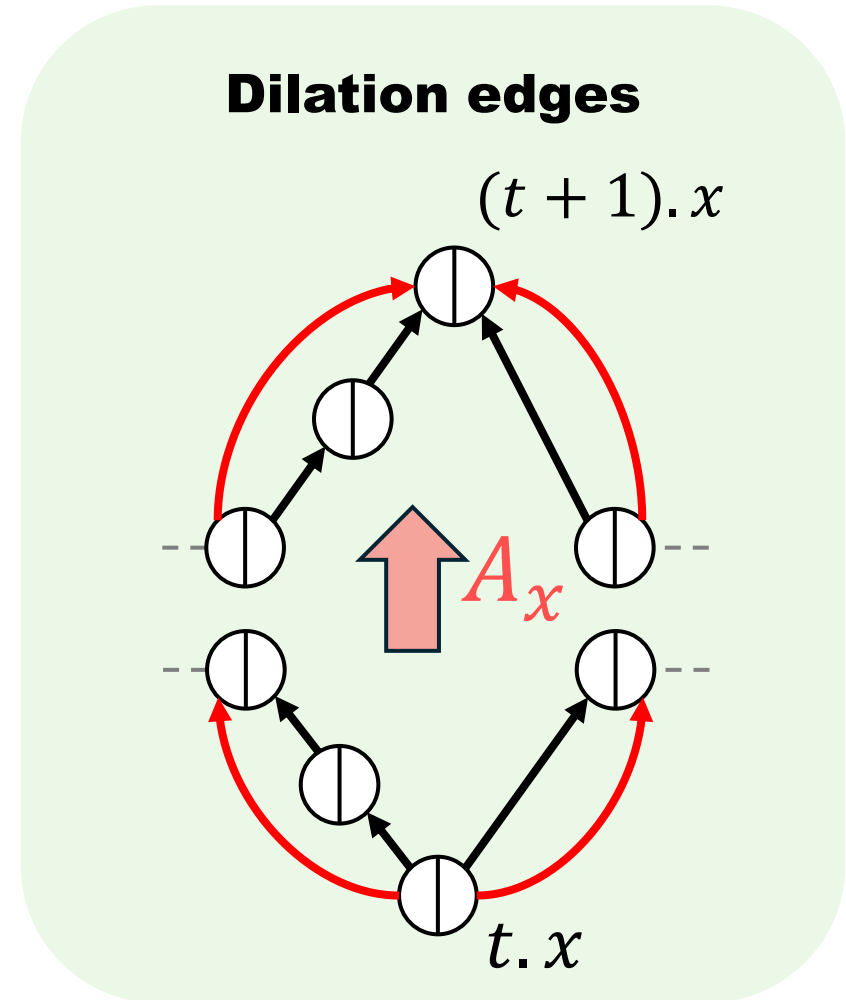
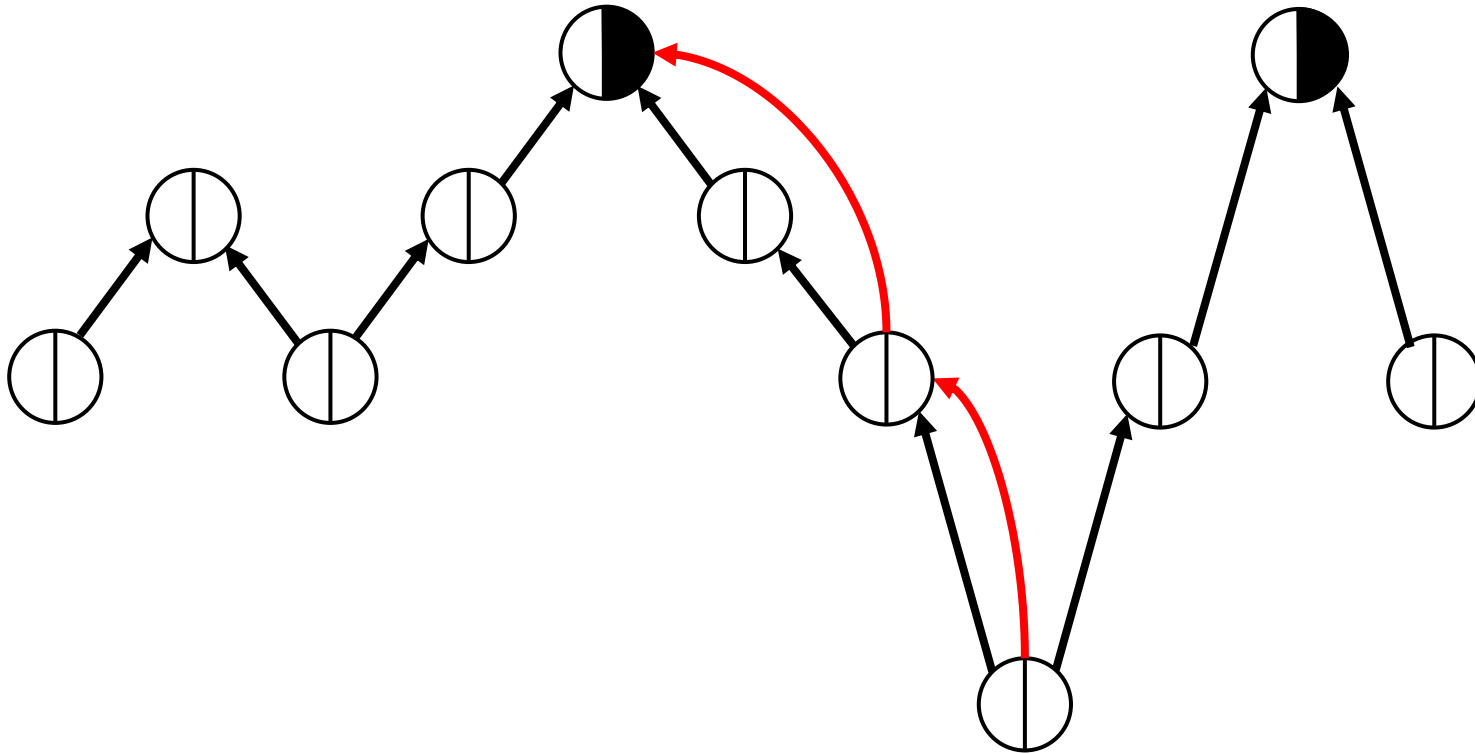
Time dilation example



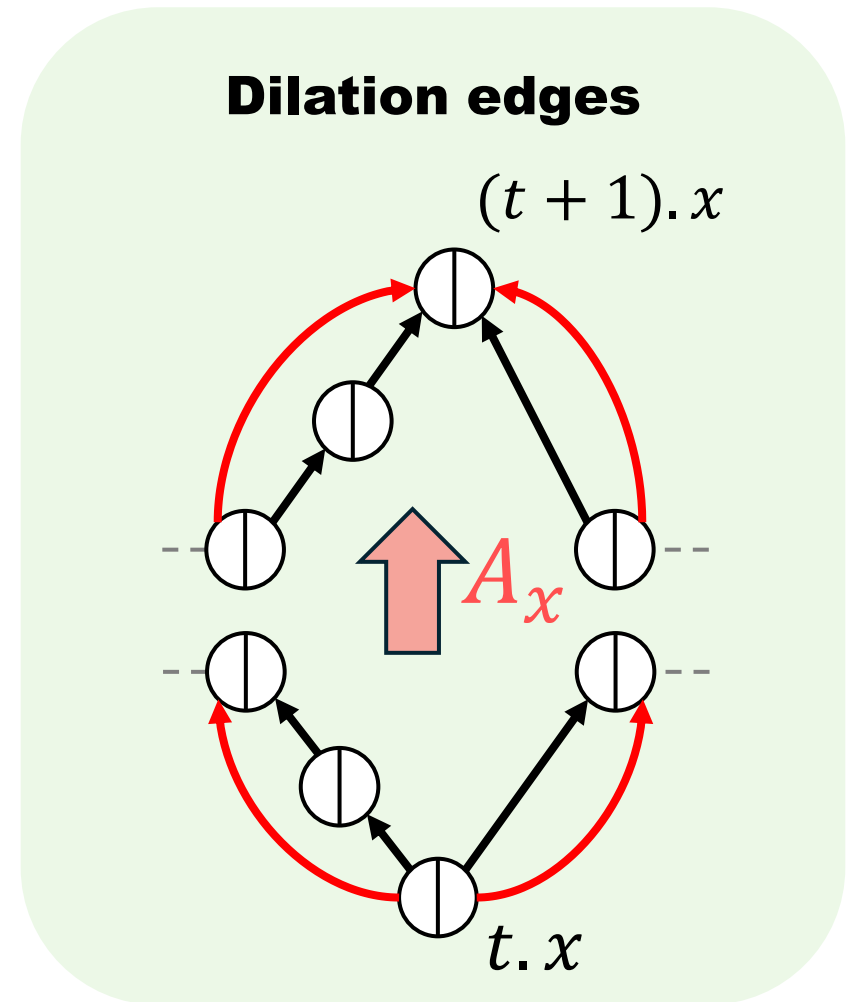
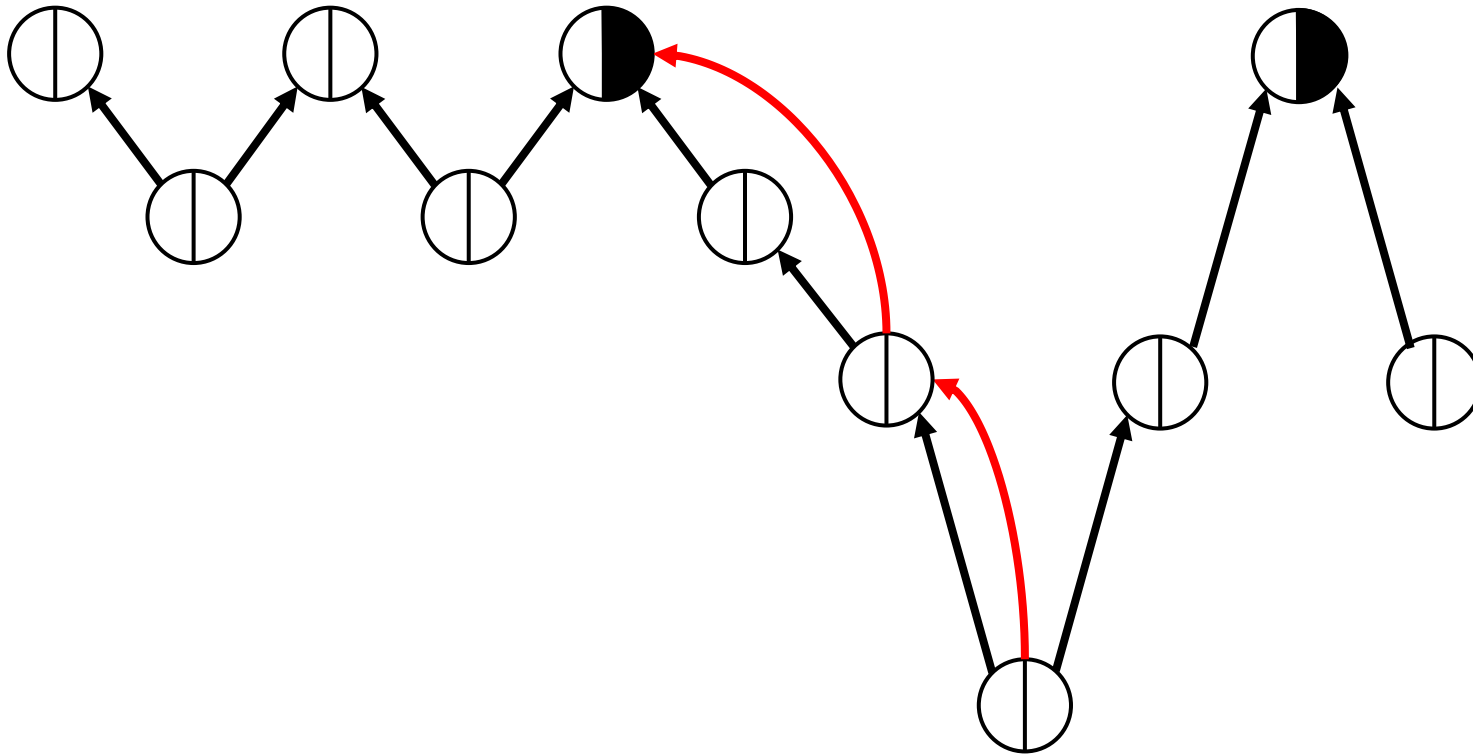
Time dilation example



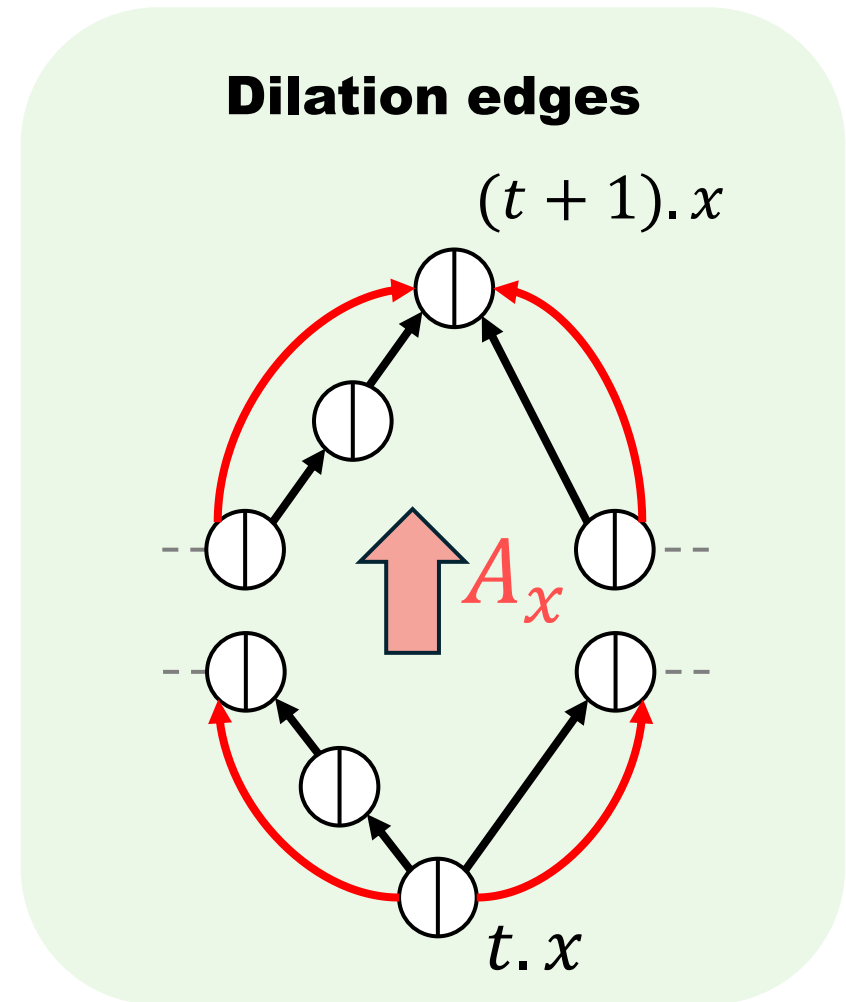
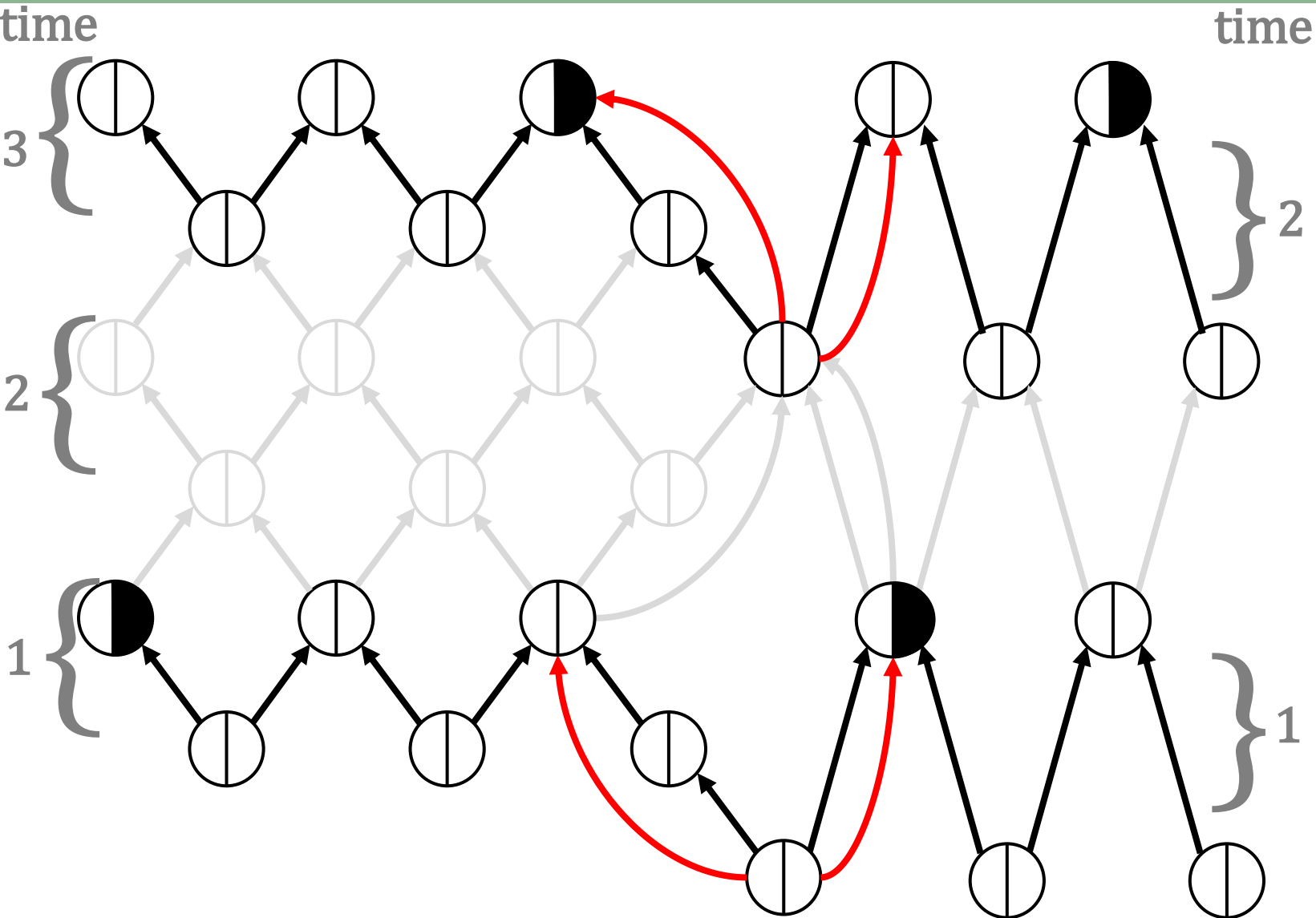
Time dilation example



Time dilation example

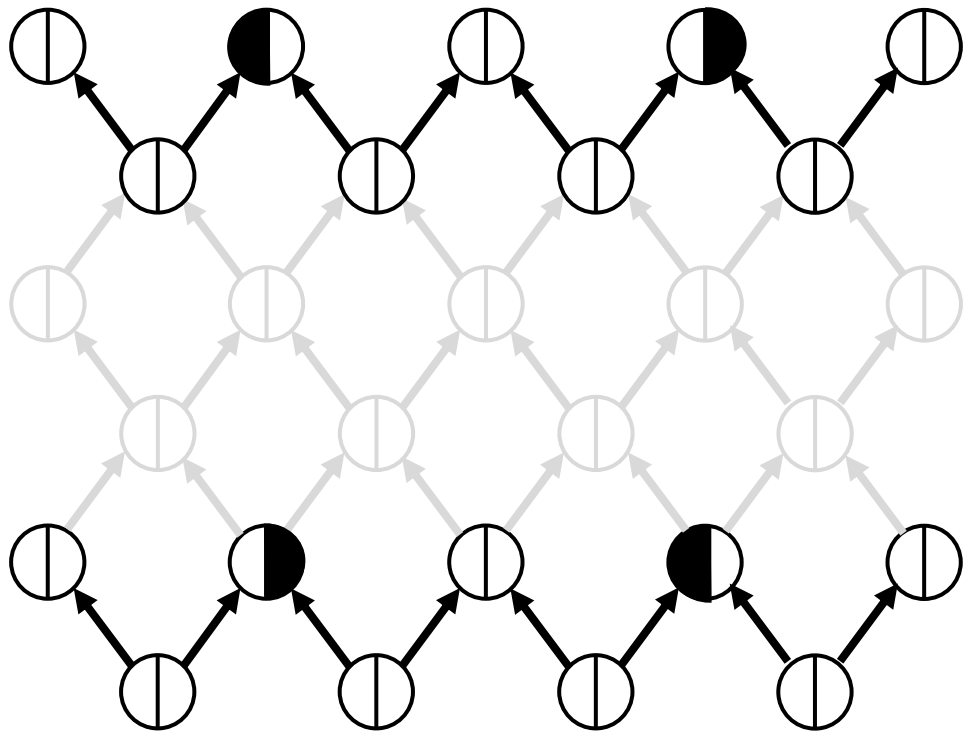


Time dilation example



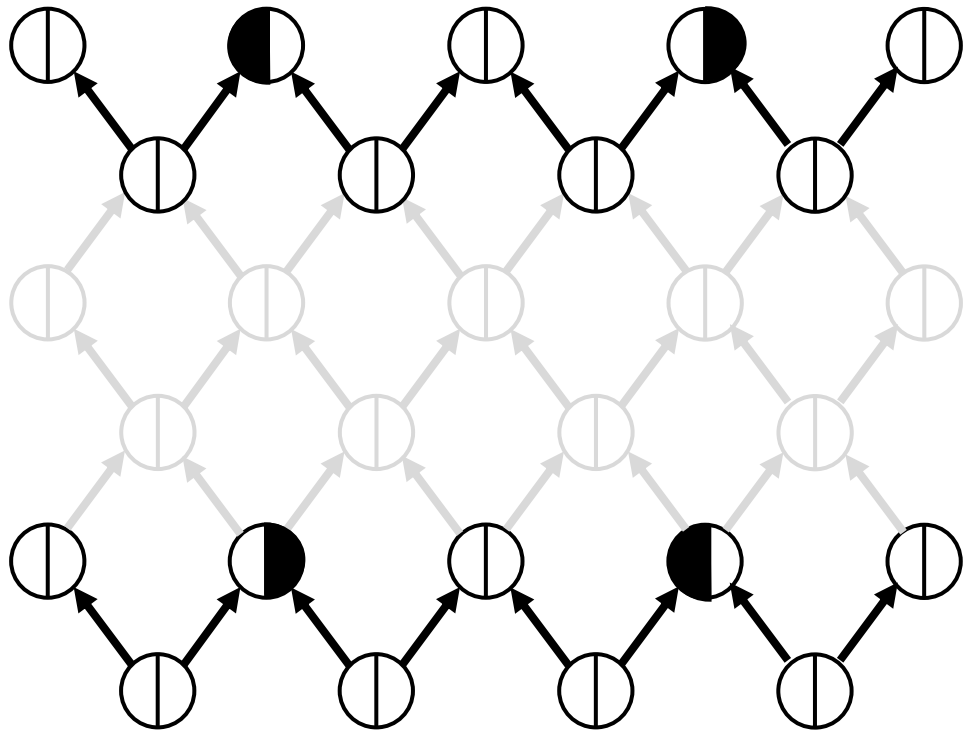
Summary

Using graph rewriting we **can simulate synchronous** dynamical system ...

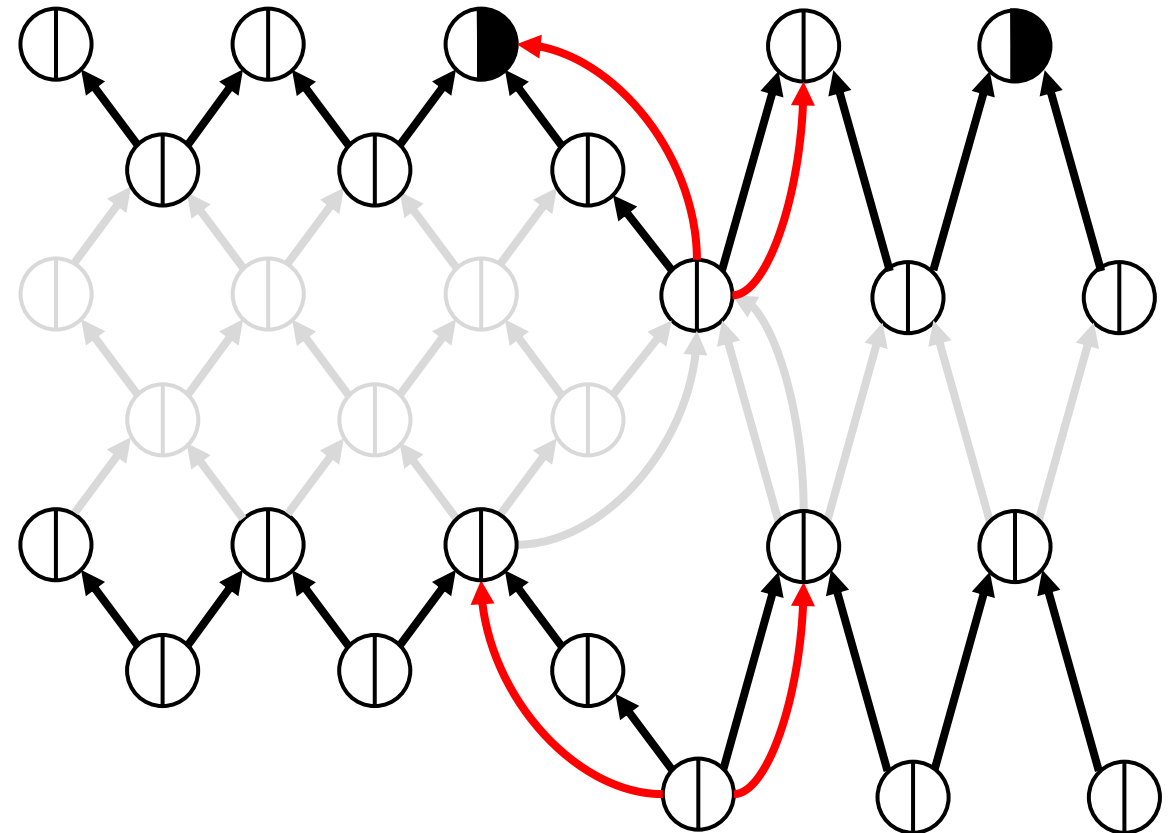


Summary

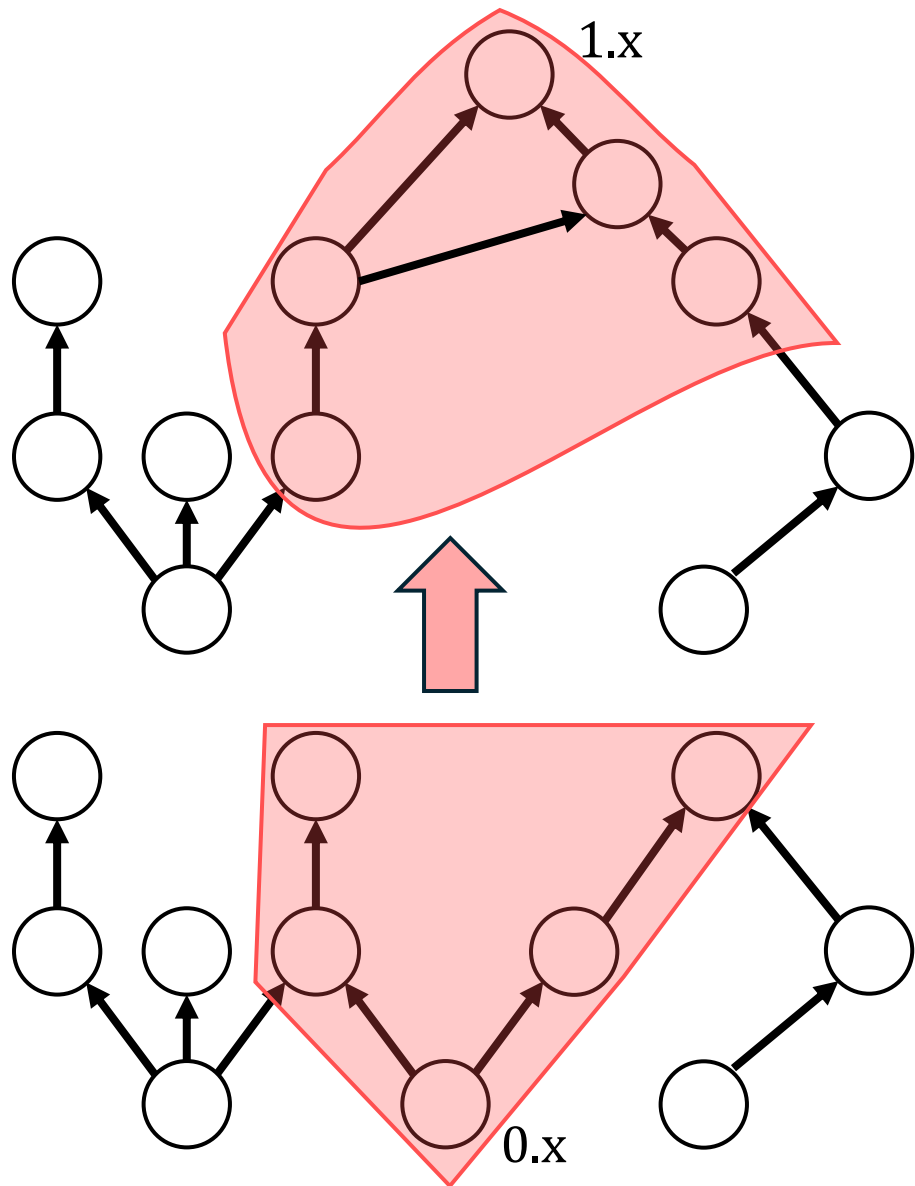
Using graph rewriting we **can simulate synchronous** dynamical system ...



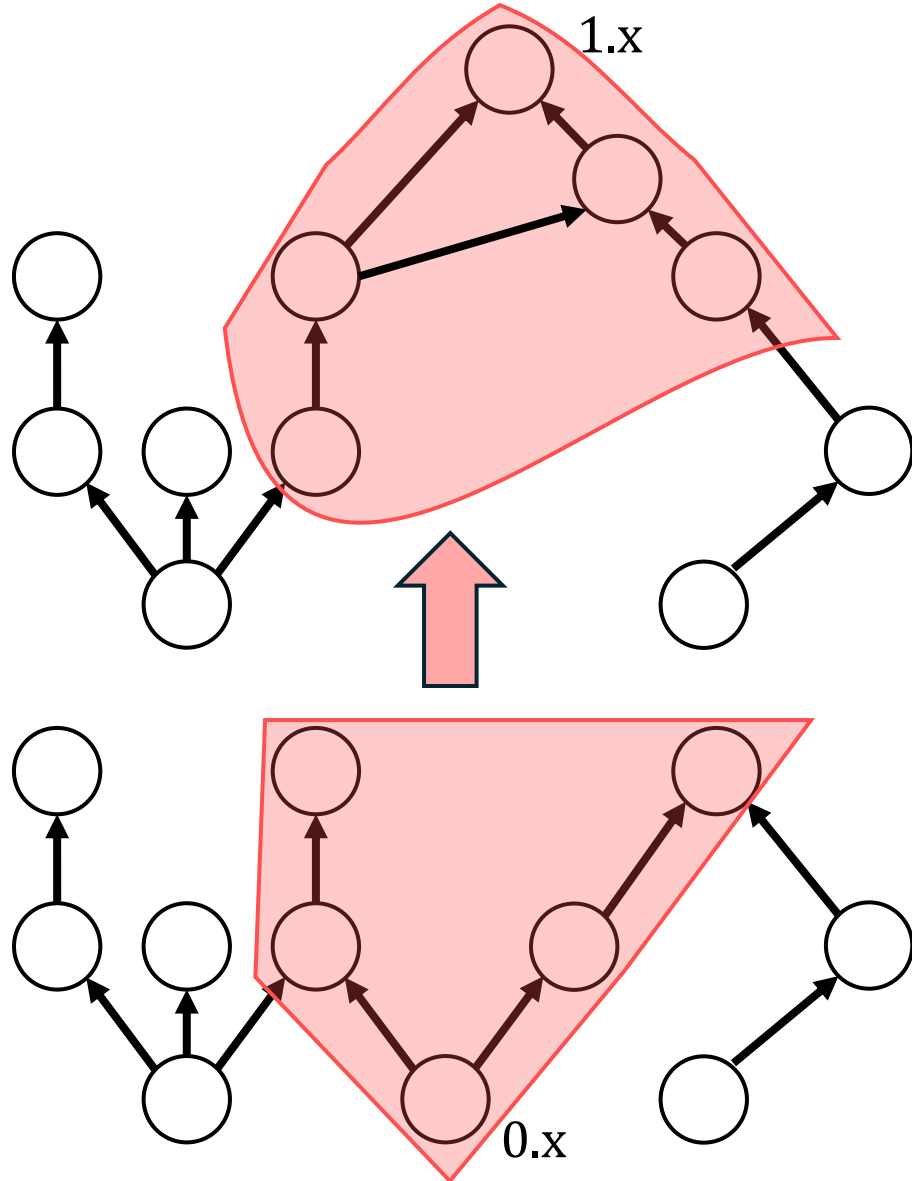
... but also represent some **intrinsically asynchronous** evolution.



General framework



General framework

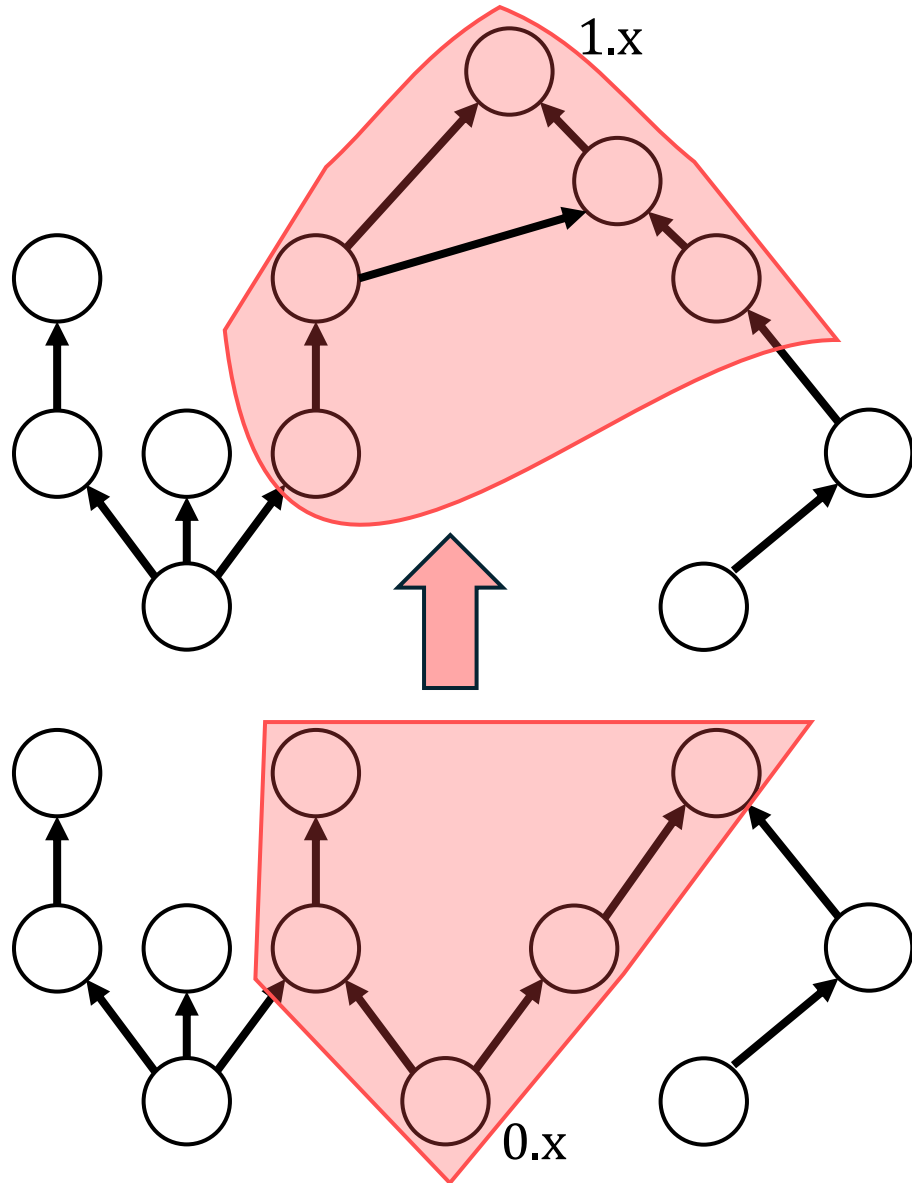


Which rewriting rules ensure that the dynamic do not depend on the rewriting strategy?

- **Confluence**



General framework

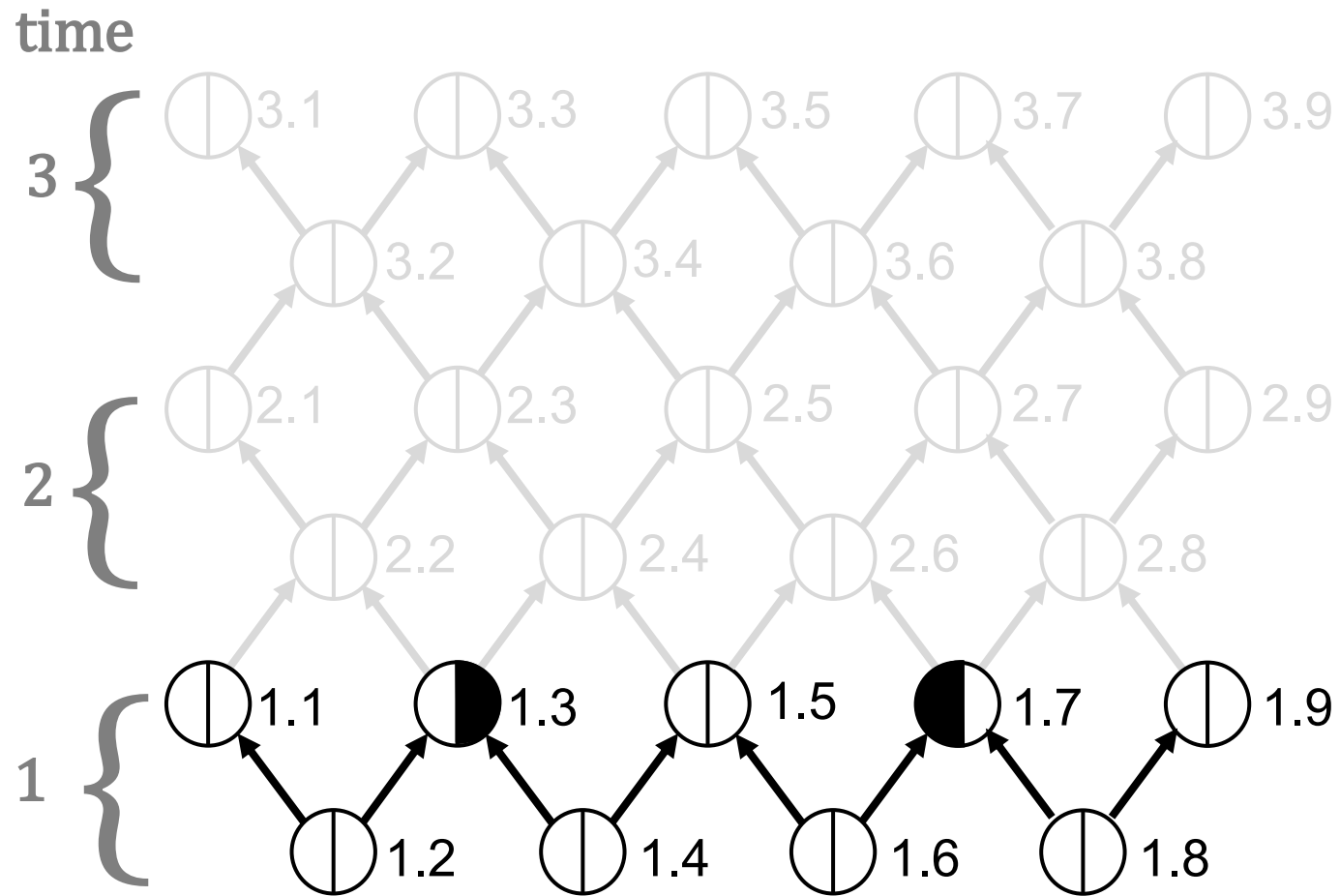


Which rewriting rules ensure that the dynamic do not depend on the rewriting strategy?

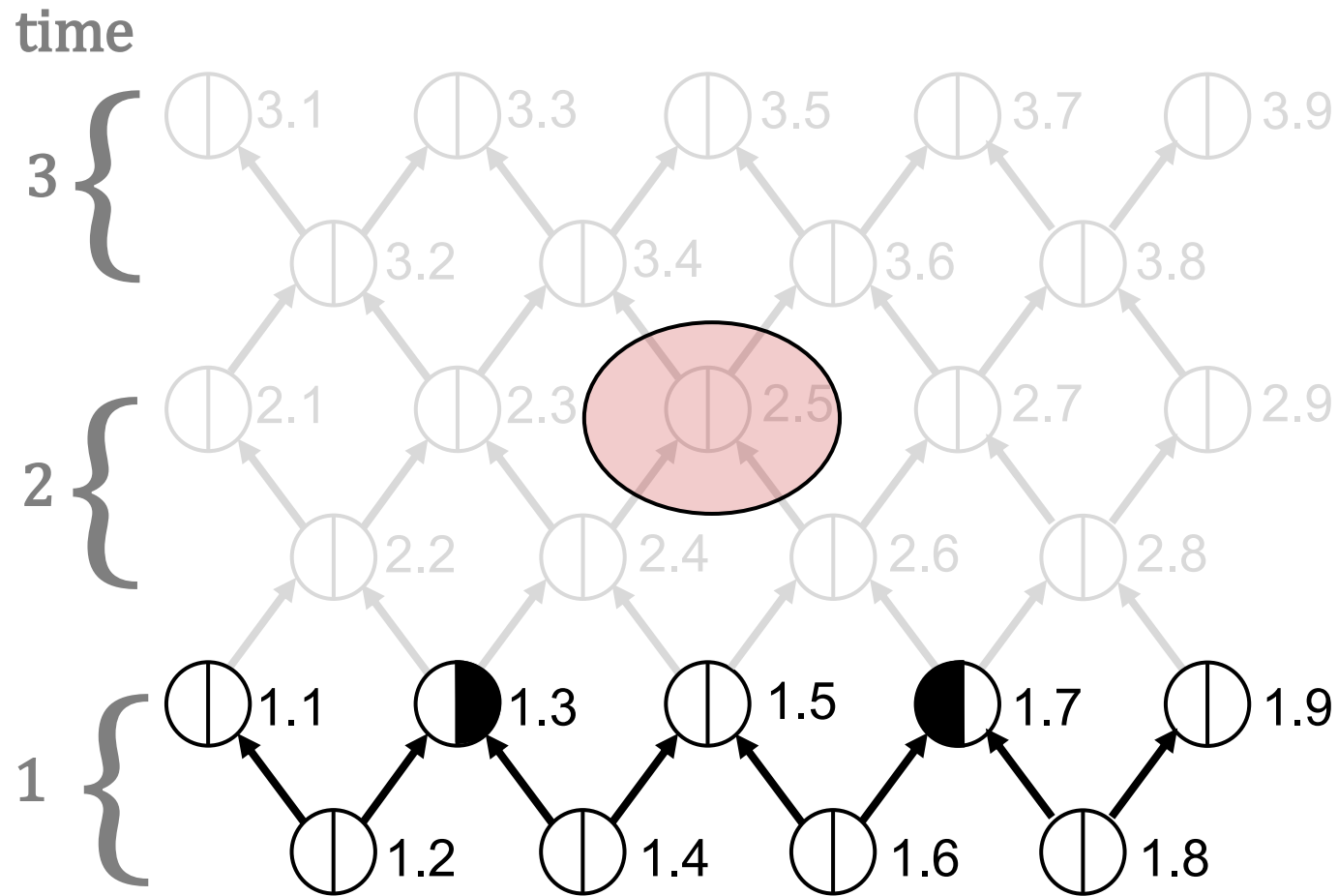
~~• **Confluence**~~

• **Space-time determinism**

A first attempt at defining determinism



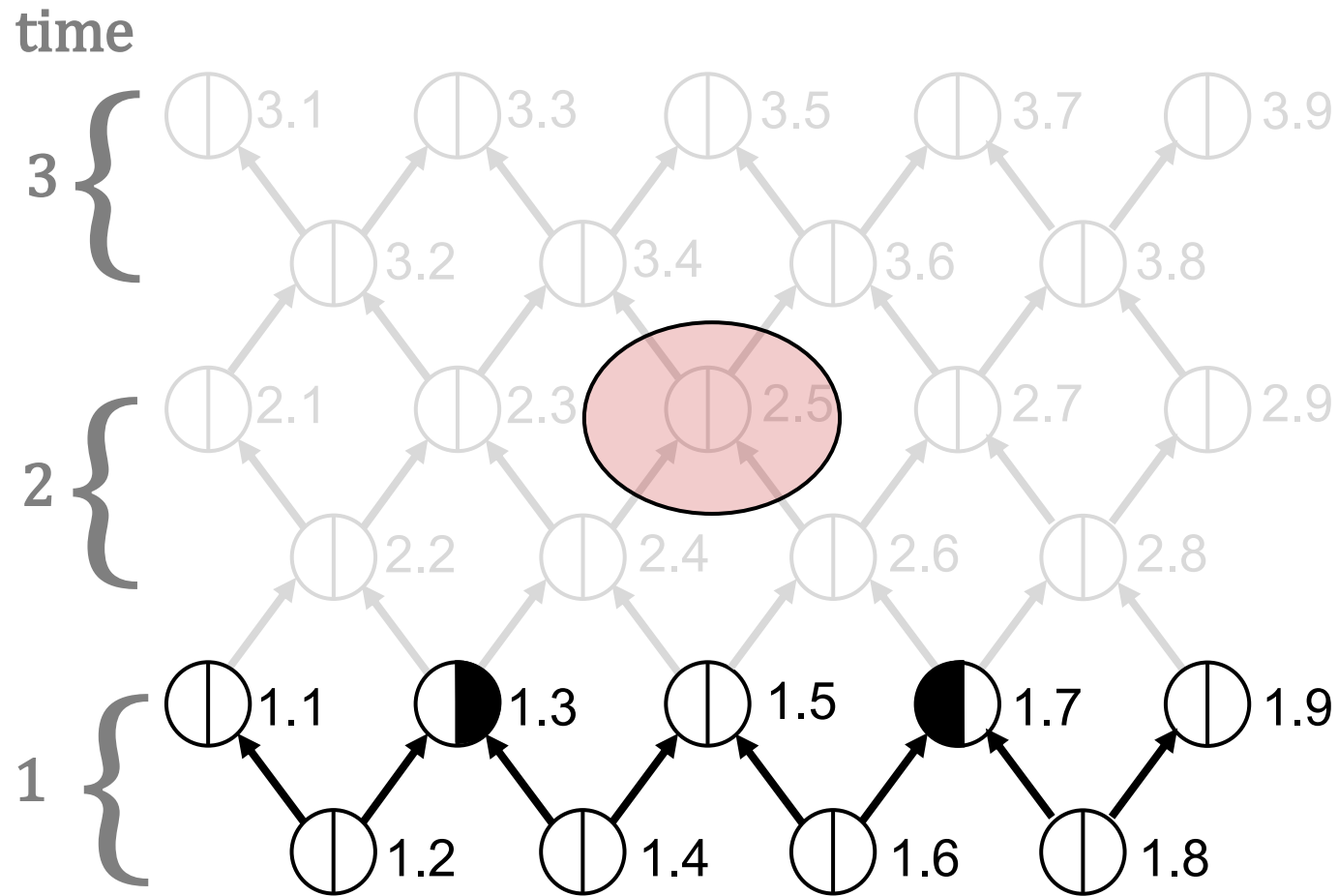
A first attempt at defining determinism



Naive determinism

The **state** of 2.5 is the **same in all cuts** passing by this node.

A first attempt at defining determinism



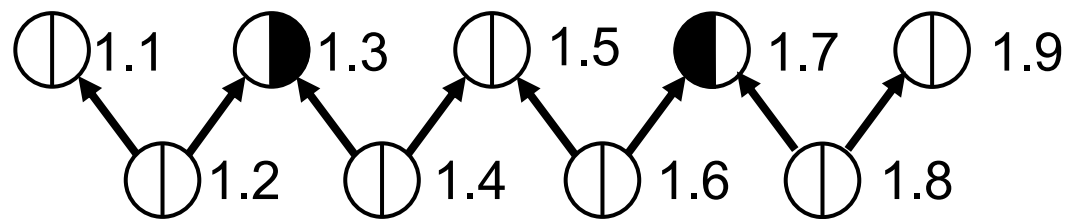
~~Naive determinism~~

~~The **state** of 2.5 is the **same** in all cuts passing by this node.~~

It should depend
on the local
shape of the cut !

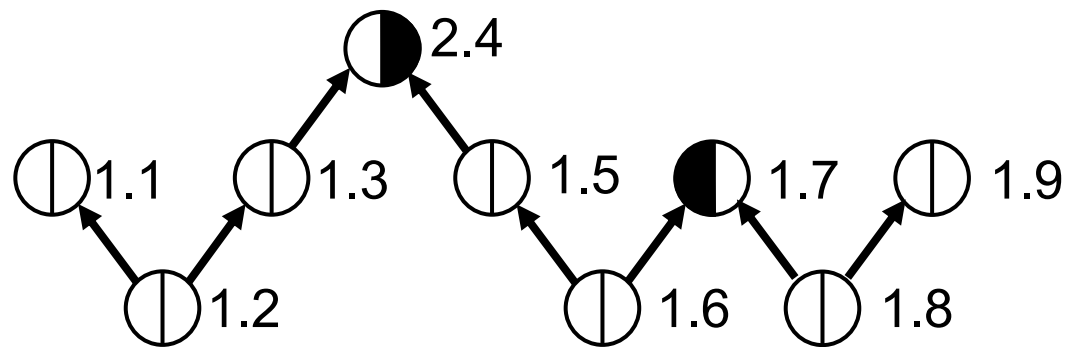
States depend on the cut

G



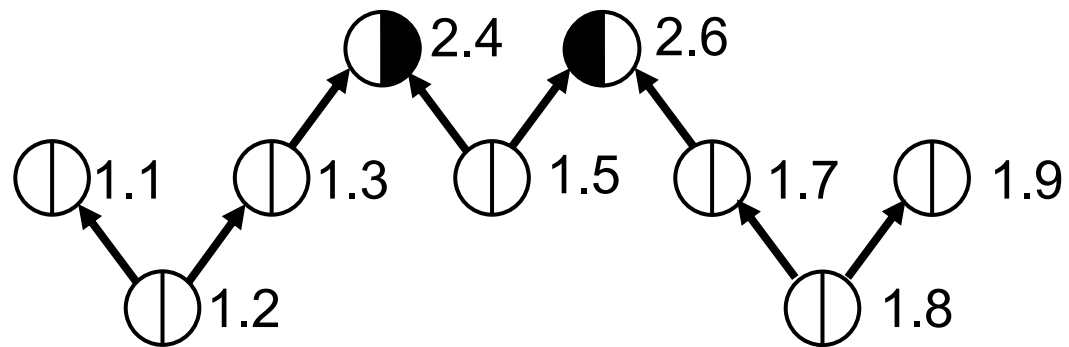
States depend on the cut

A_4G



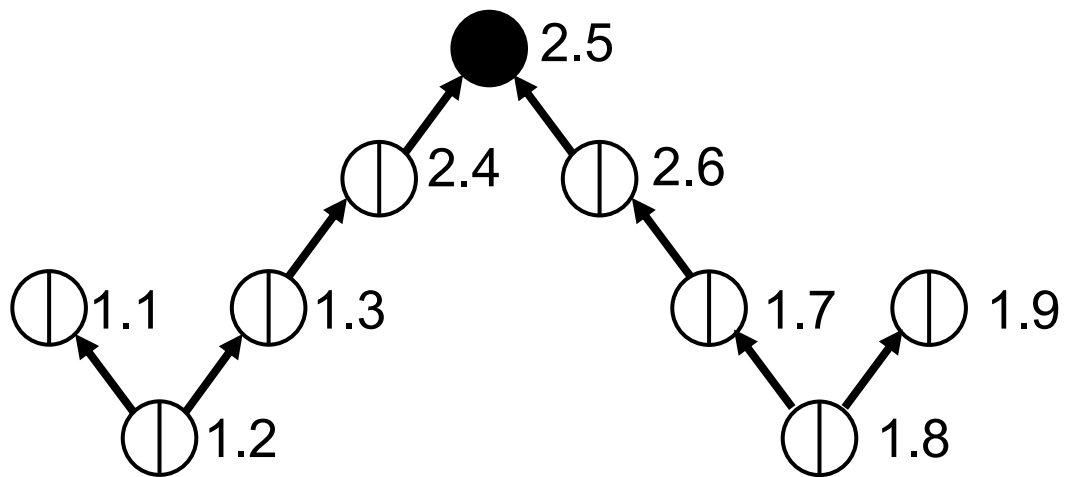
States depend on the cut

$A_6 A_4 G$



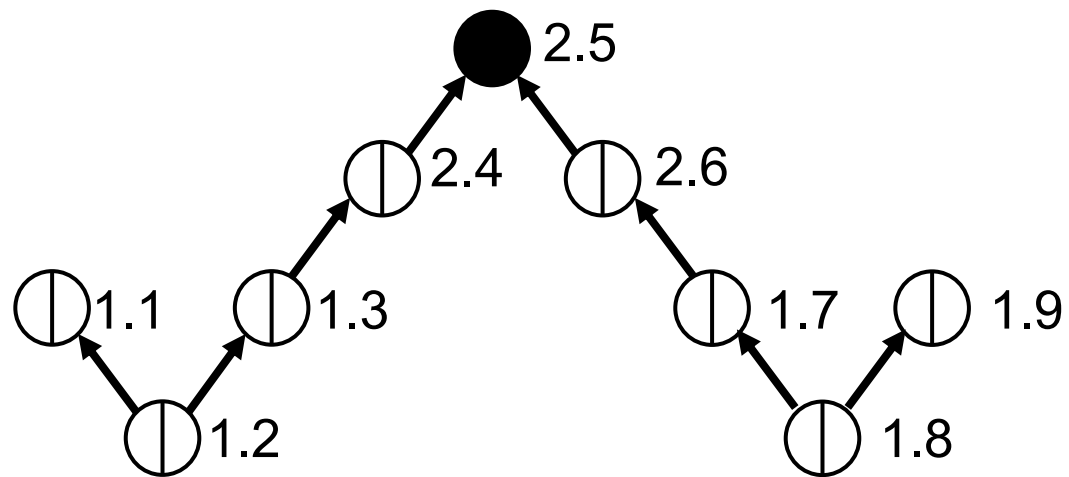
States depend on the cut

$A_5 A_6 A_4 G$

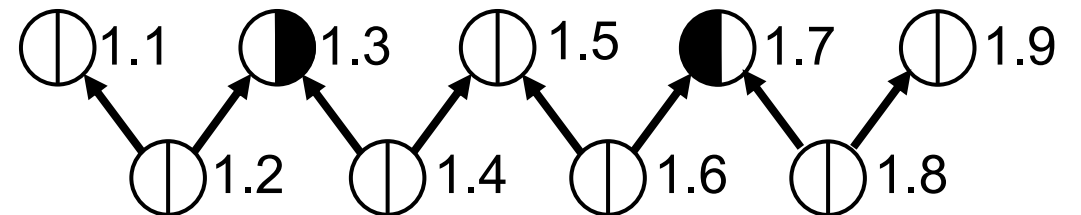


States depend on the cut

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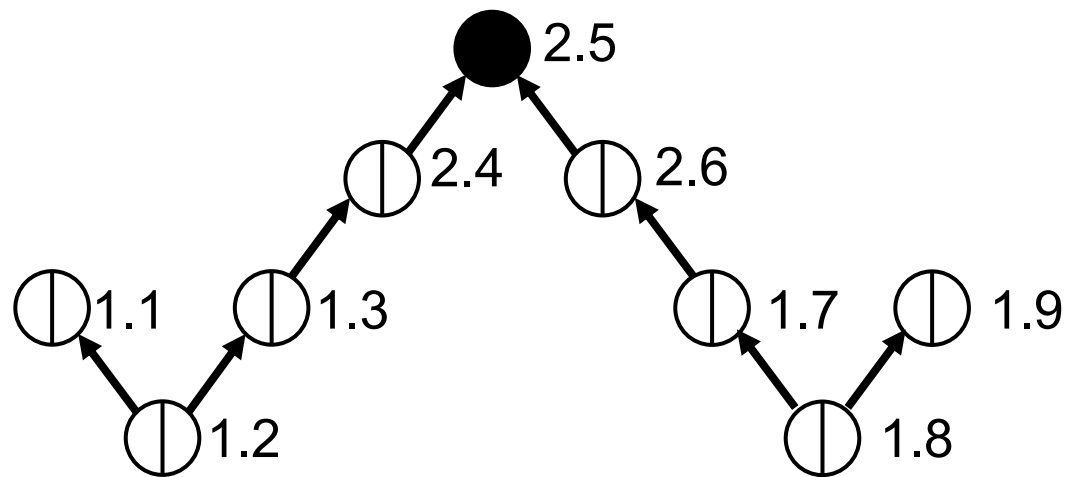


G

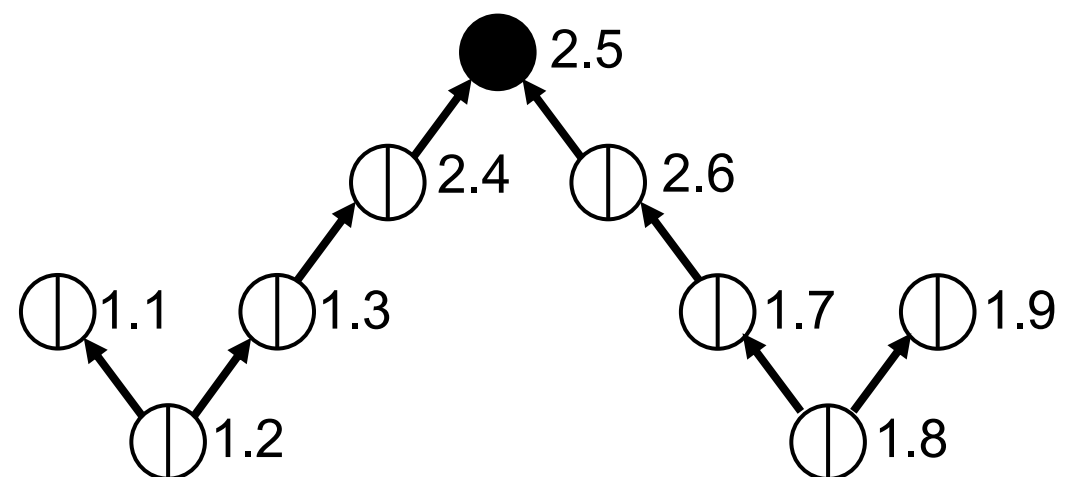


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$A_5 A_6 A_4 G$

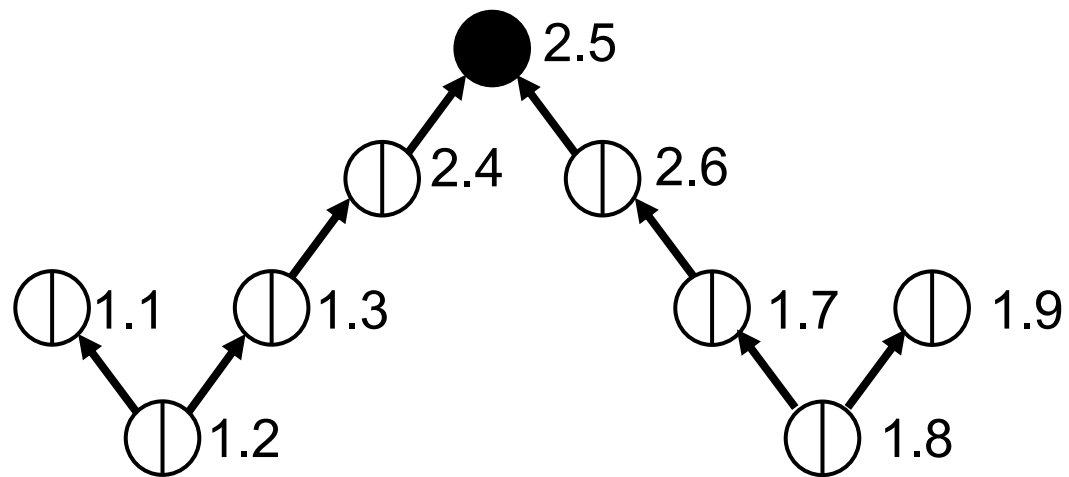


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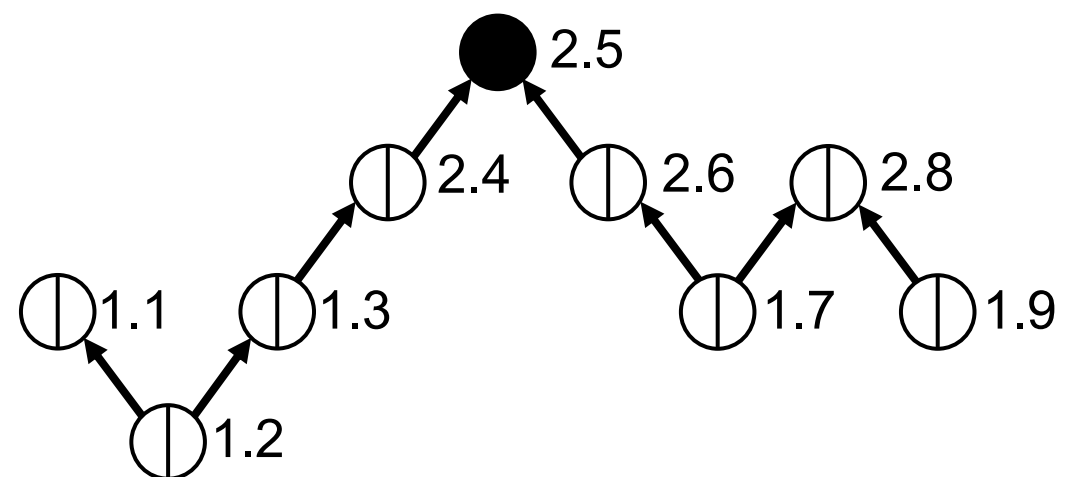


States depend on the cut

$A_5 A_6 A_4 G$

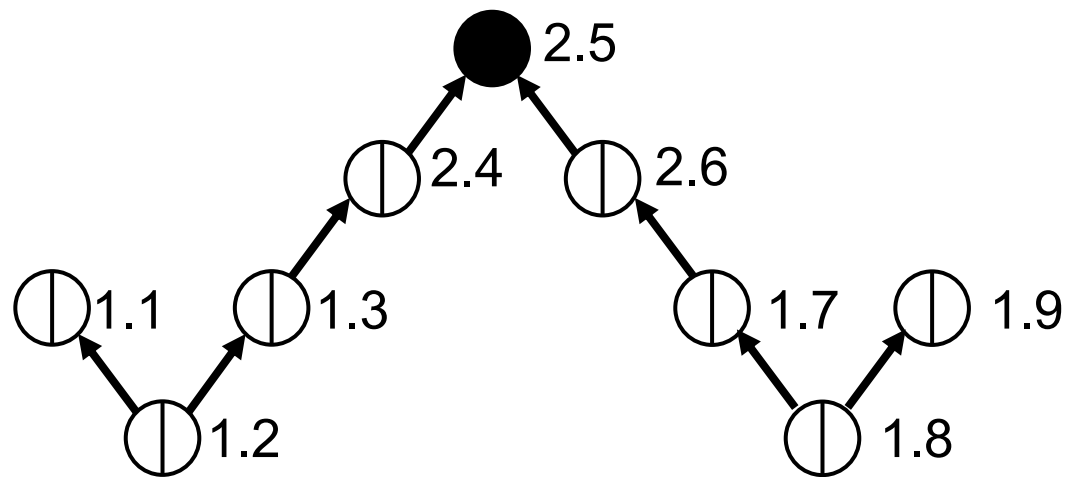


$A_8 A_5 A_6 A_4 G$

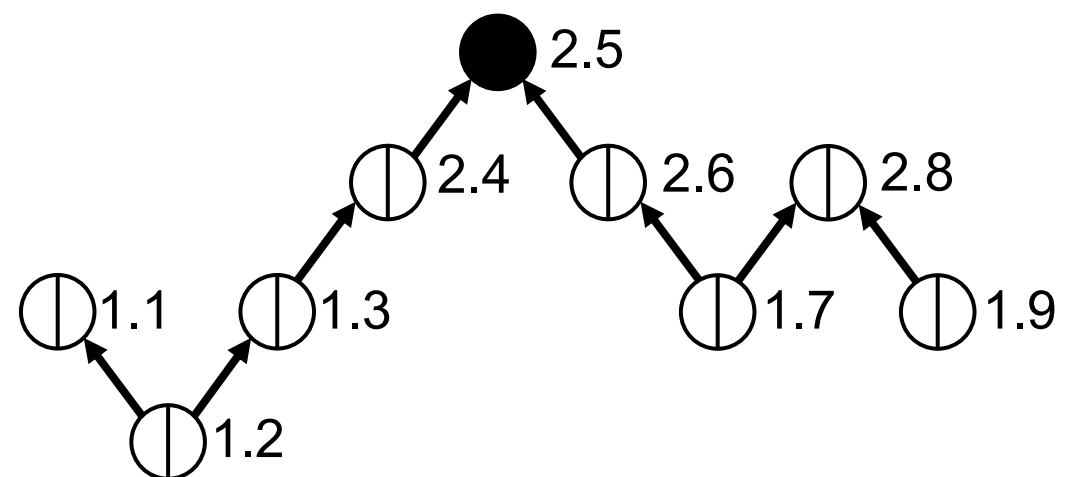


States depend on the cut

$A_5 A_6 A_4 G$

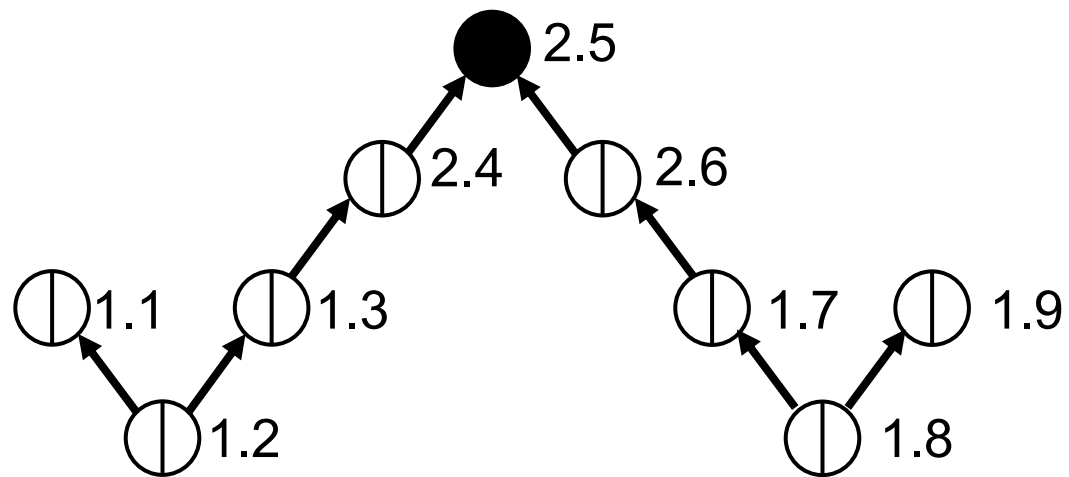


$A_8 A_5 A_6 A_4 G$

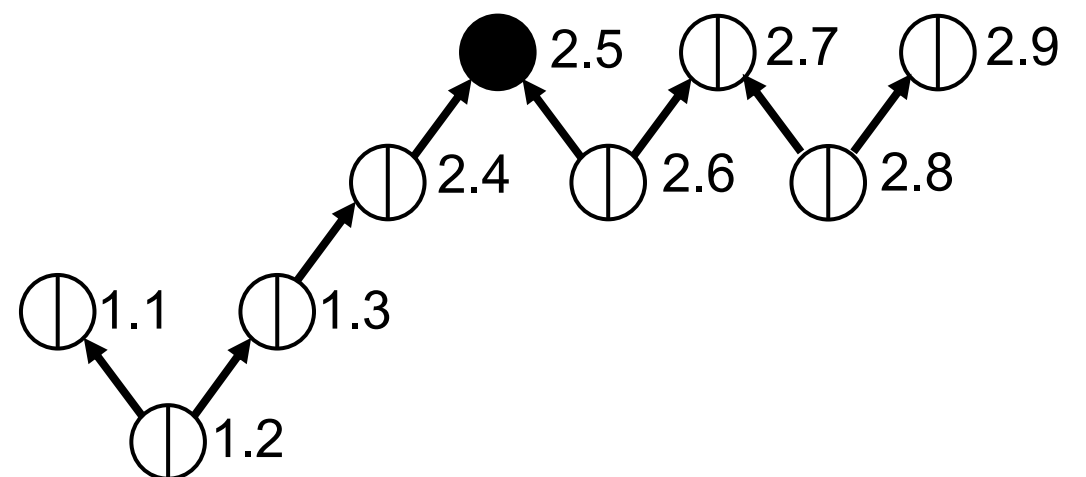


States depend on the cut

$A_5 A_6 A_4 G$

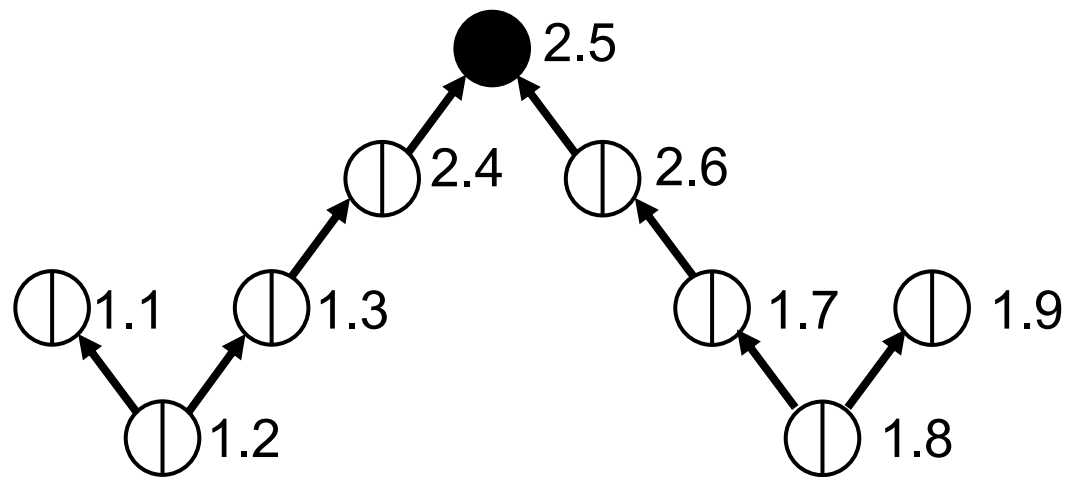


$A_9 A_7 A_8 A_5 A_6 A_4 G$

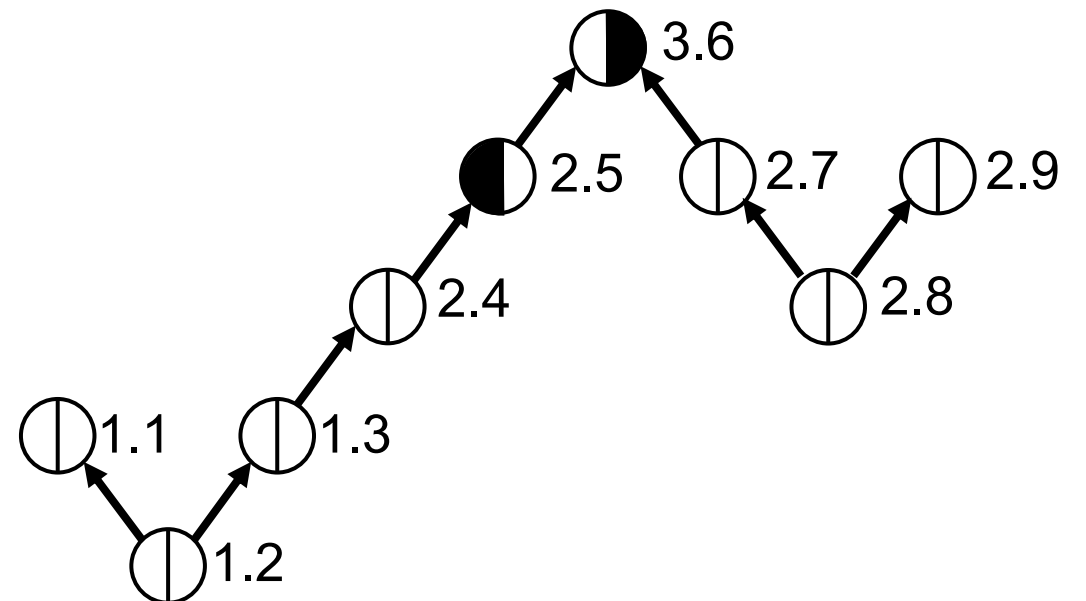


States depend on the cut

$A_5 A_6 A_4 G$

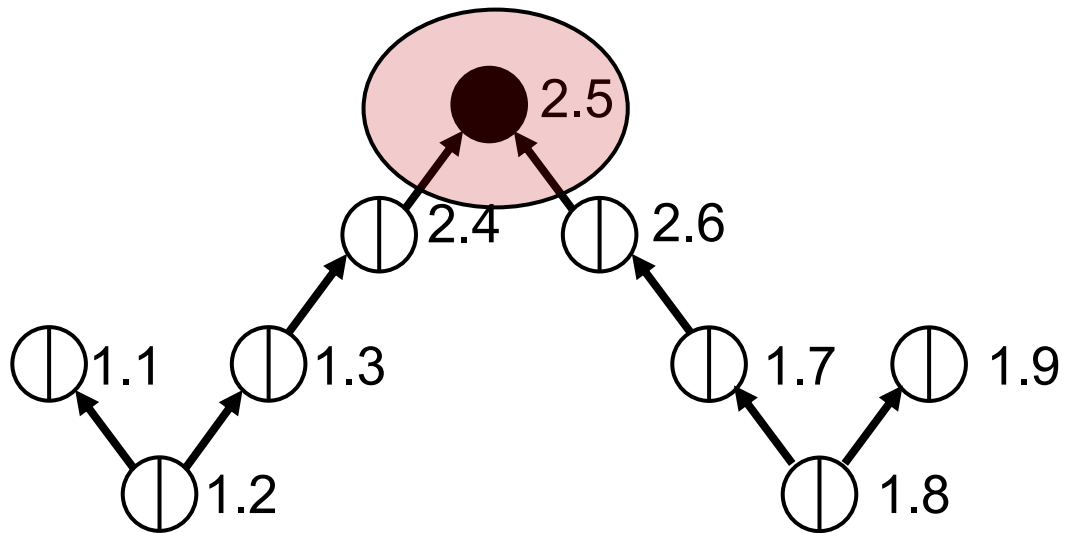


$A_6 A_9 A_8 A_5 A_4 G$



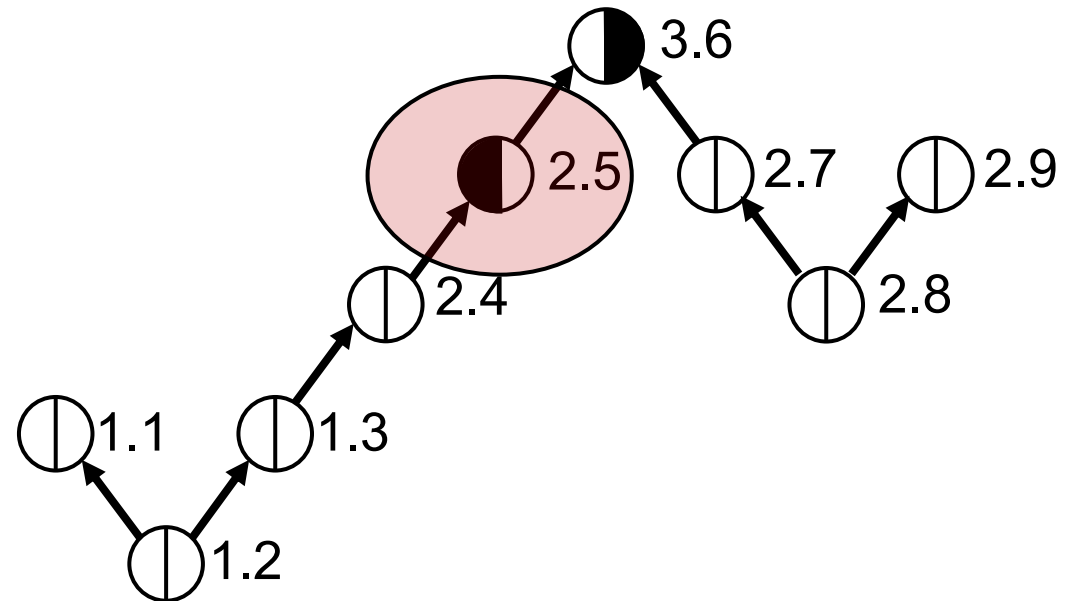
States depend on the cut

$A_5 A_6 A_4 G$



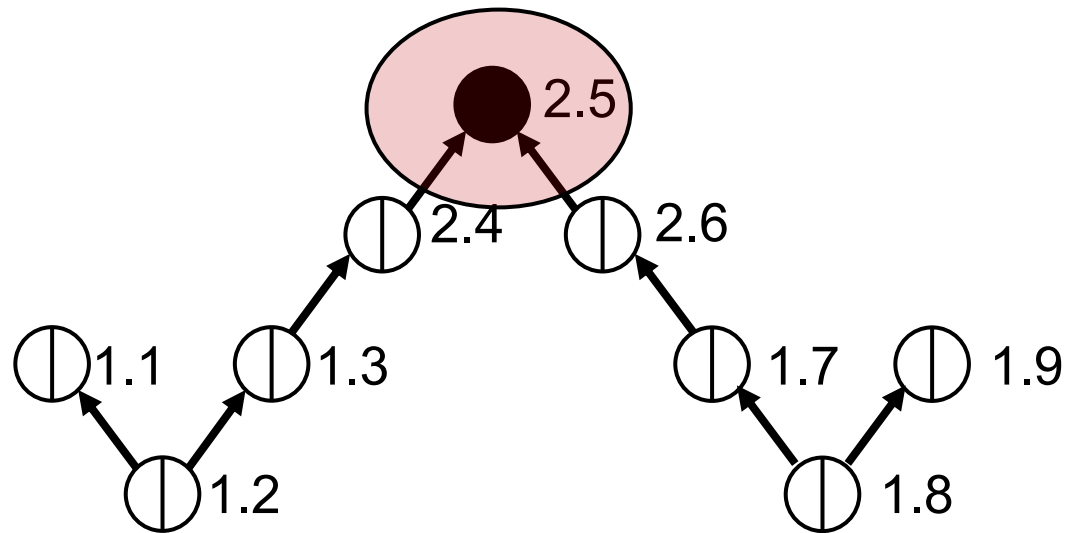
\neq

$A_6 A_9 A_7 A_8 A_5 A_6 A_4 G$



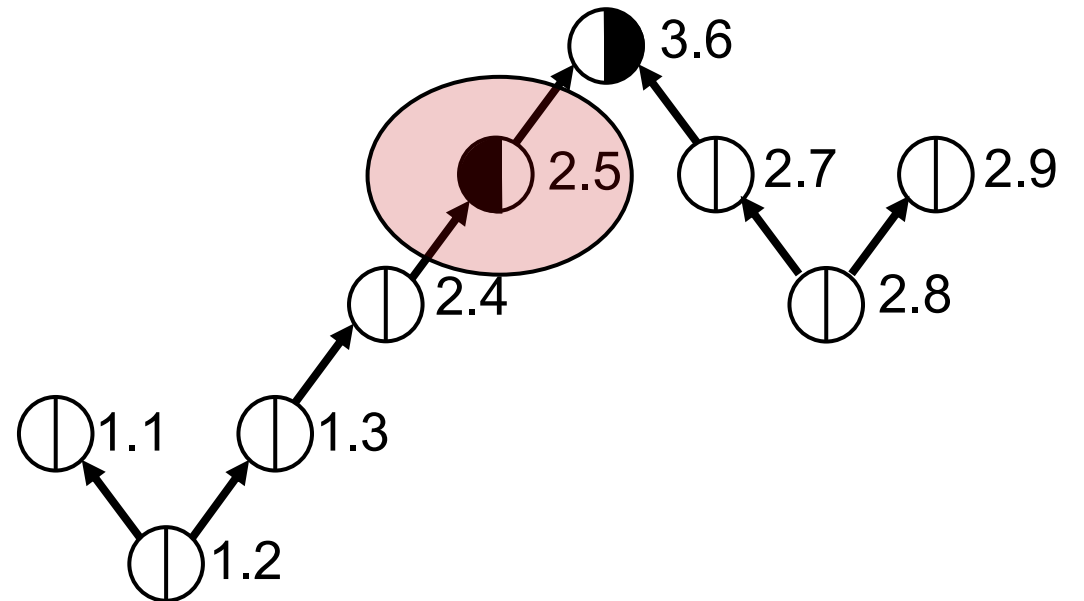
States depend on the cut

$$A_5 A_6 A_4 G = A_w$$

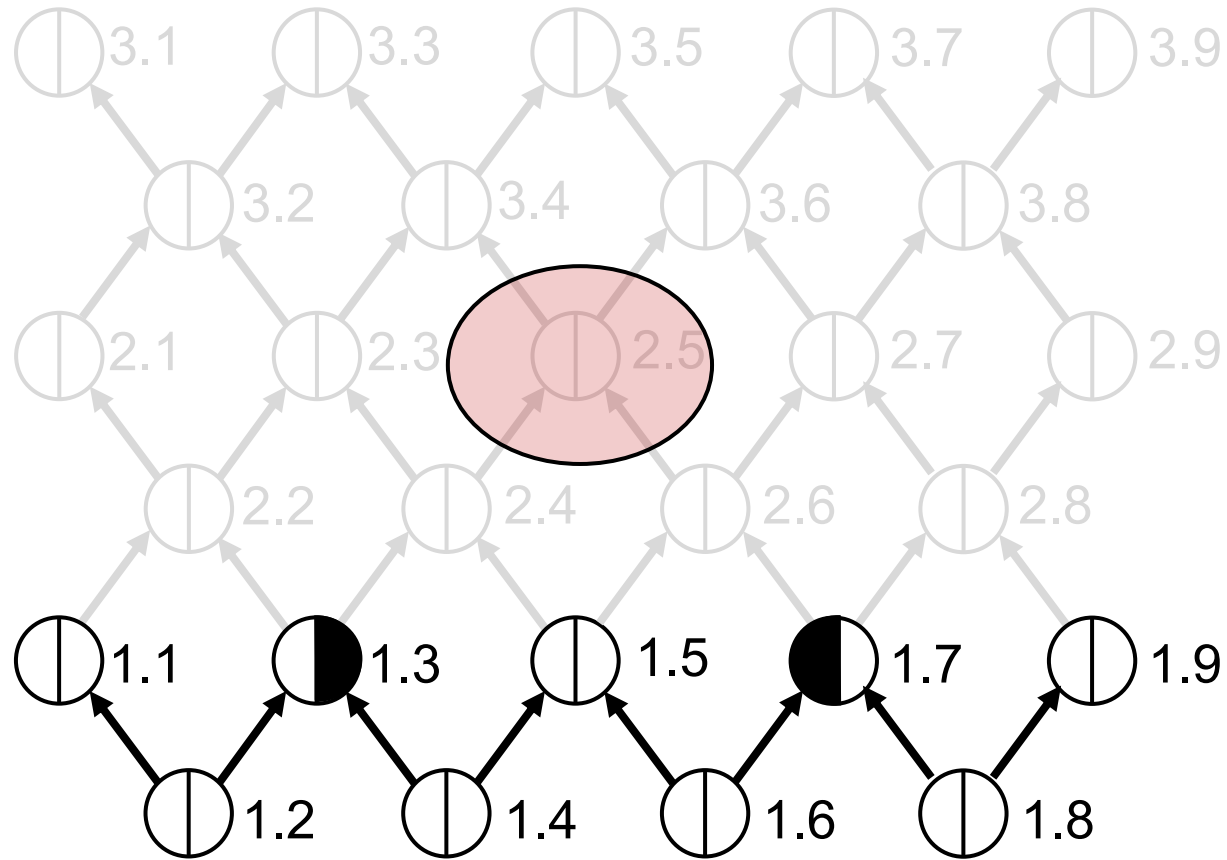


≠

$$A_6 A_9 A_7 A_8 A_5 A_4 G = A_w'$$



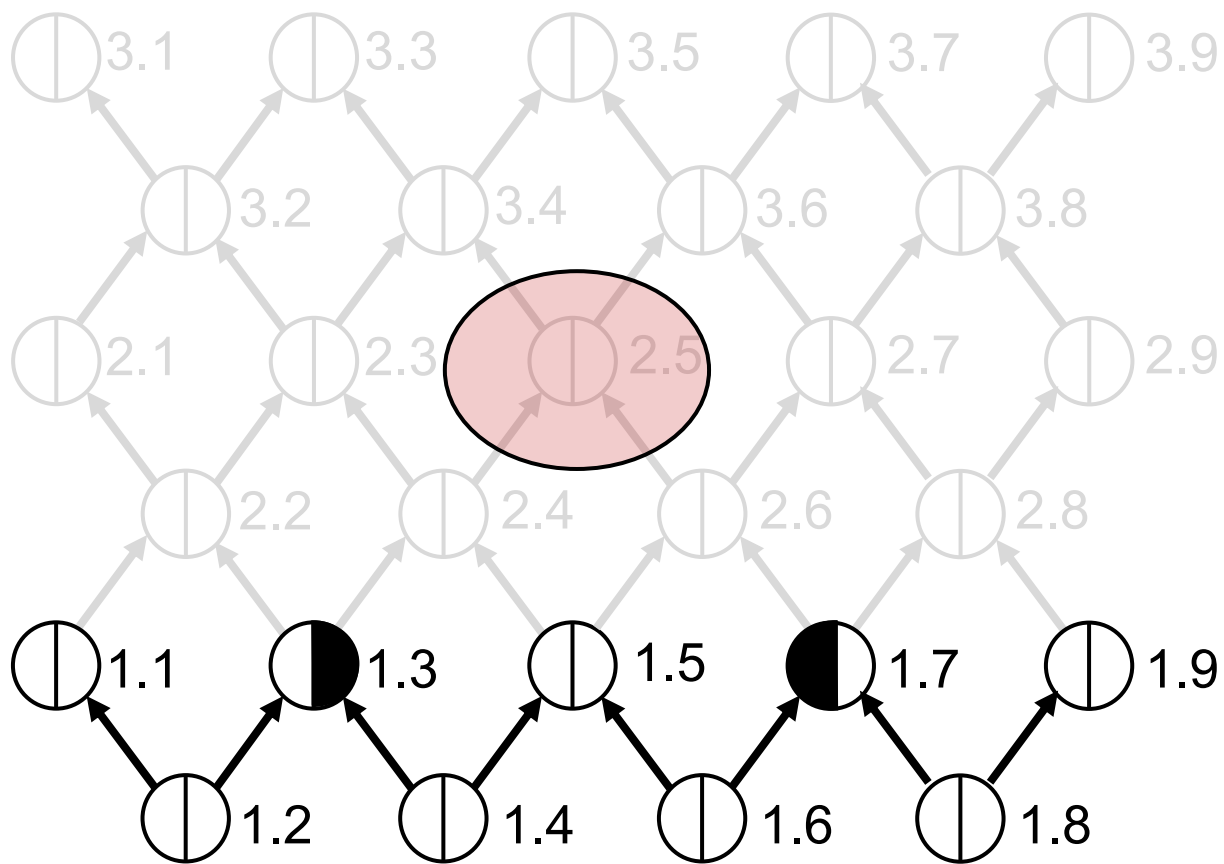
Space-time Determinism



Space-time Determinism

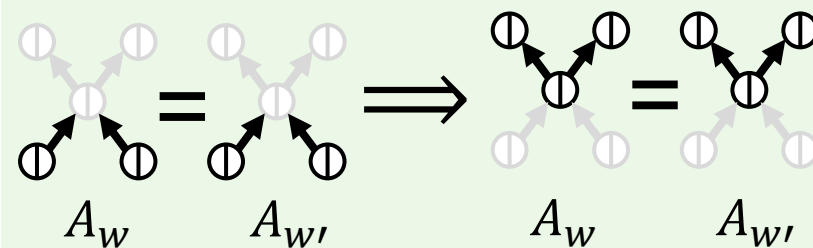
The **state** of 2.5 is the **same for 2 locally identical cuts** passing by this node.

Space-time Determinism



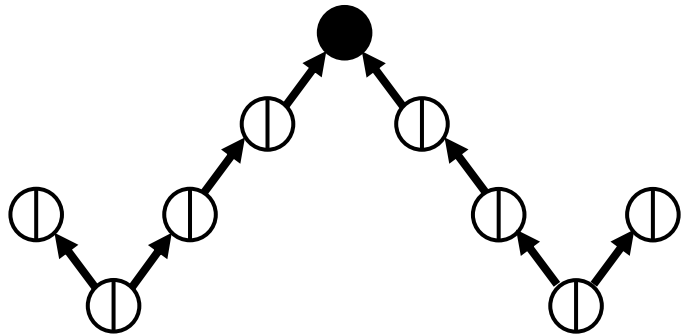
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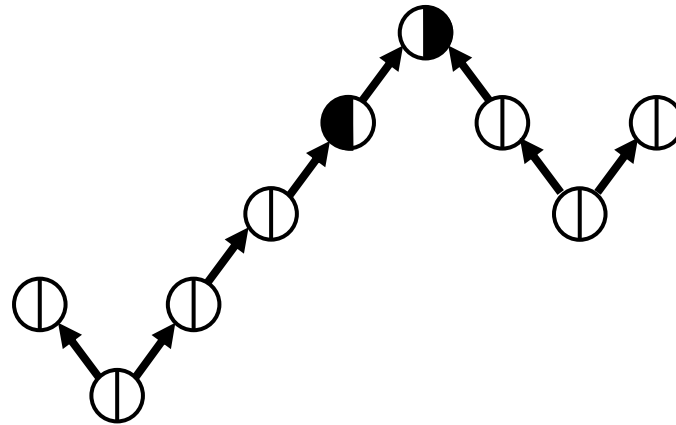


Space-time Determinism

$A_w G$

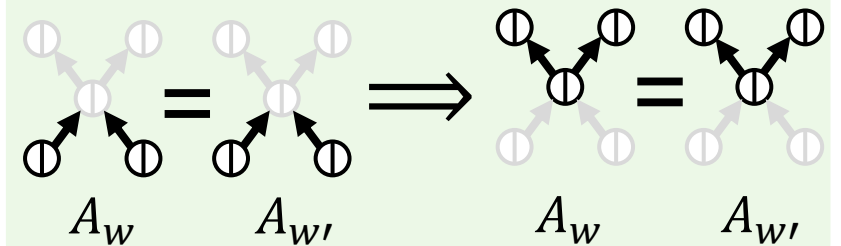


$A_{w'} G$



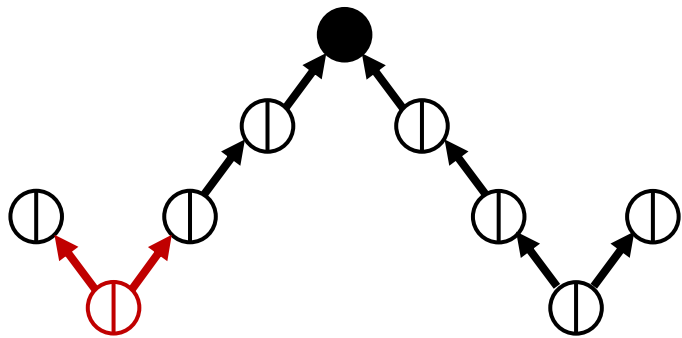
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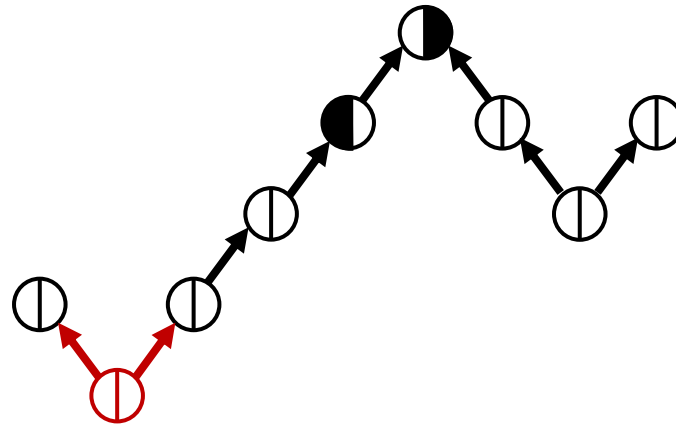


Space-time Determinism

$A_w G$

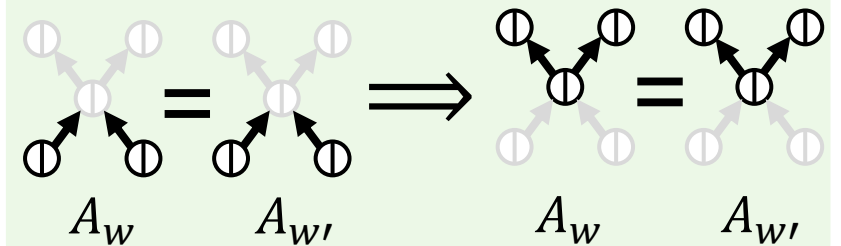


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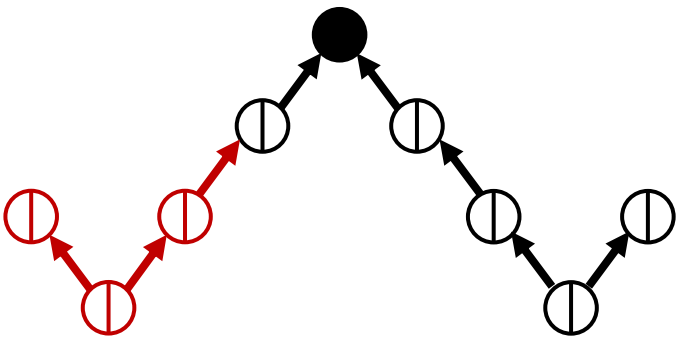
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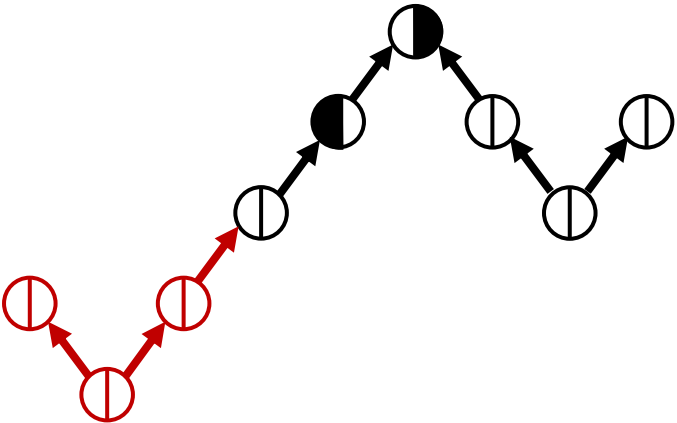


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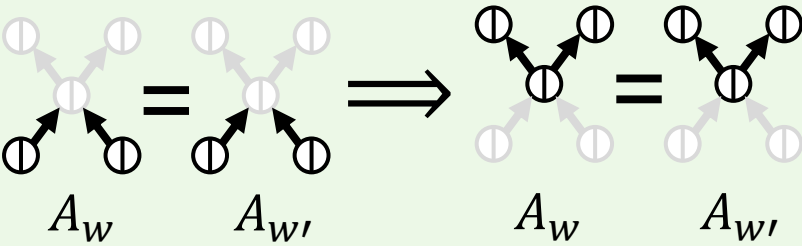


$A_{w'} G$



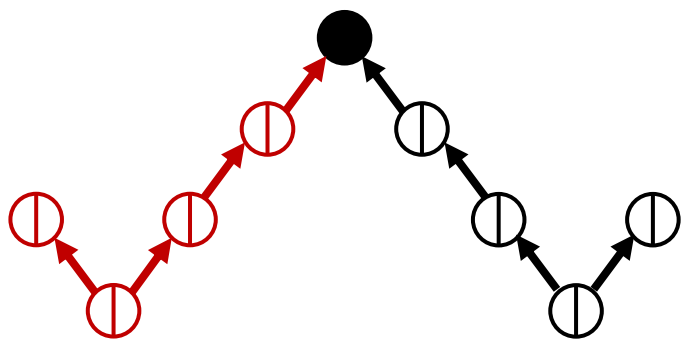
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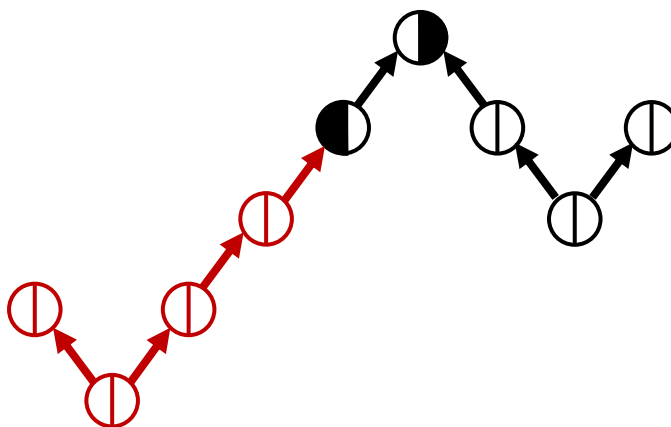


Space-time Determinism

$A_w G$

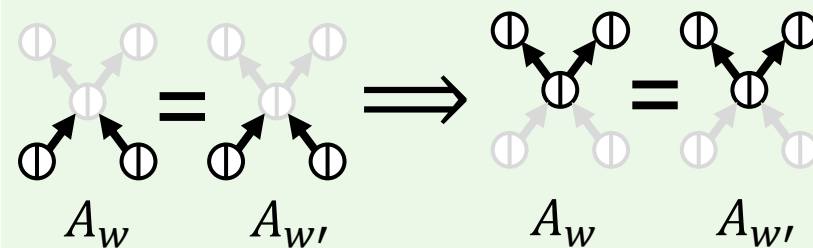


$A_{w'} G$



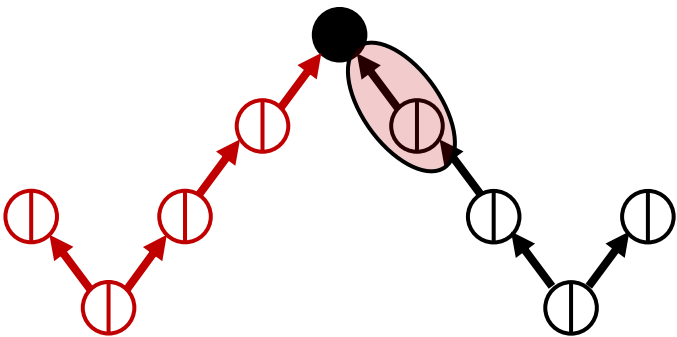
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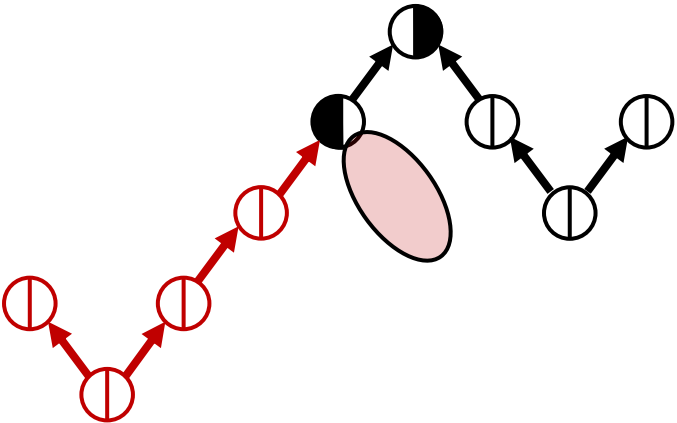


Space-time Determinism

$A_w G$

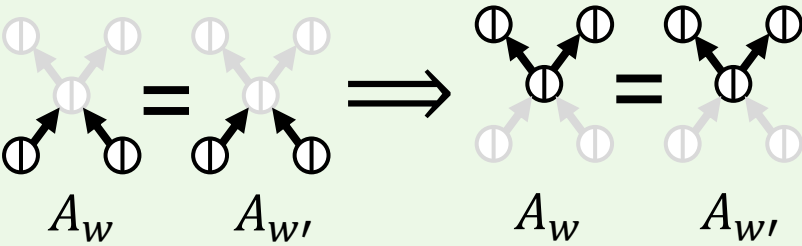


$A_{w'} G$



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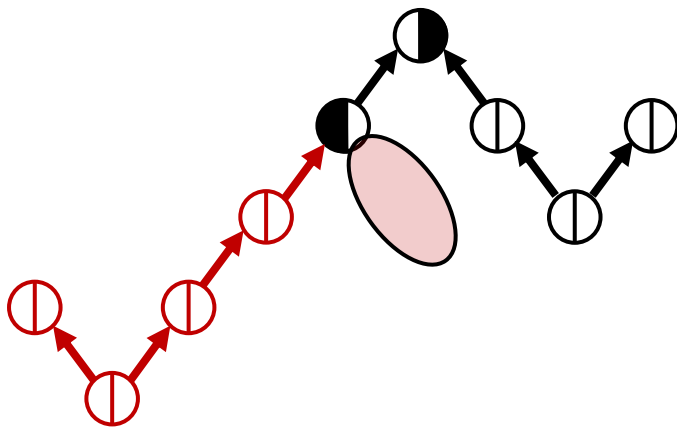
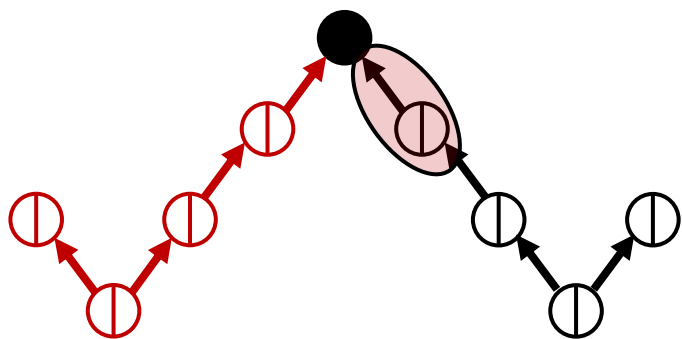


Space-time Determinism

$A_w G$

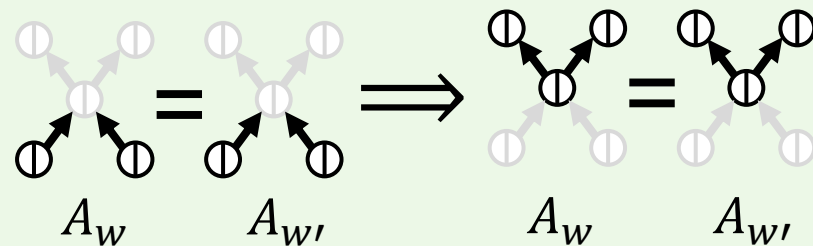
||

$A_{w'} G$



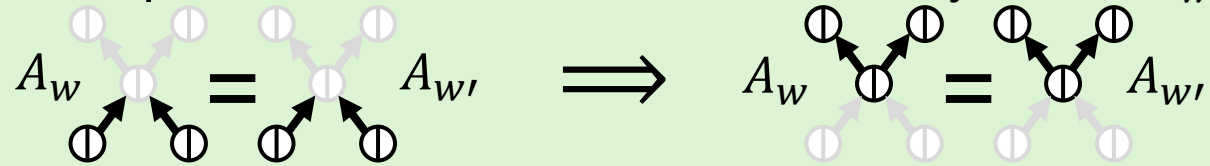
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Ensuring determinism

Goal : prove A deterministic, i.e. we always have $A_w \parallel A_{w'}$:

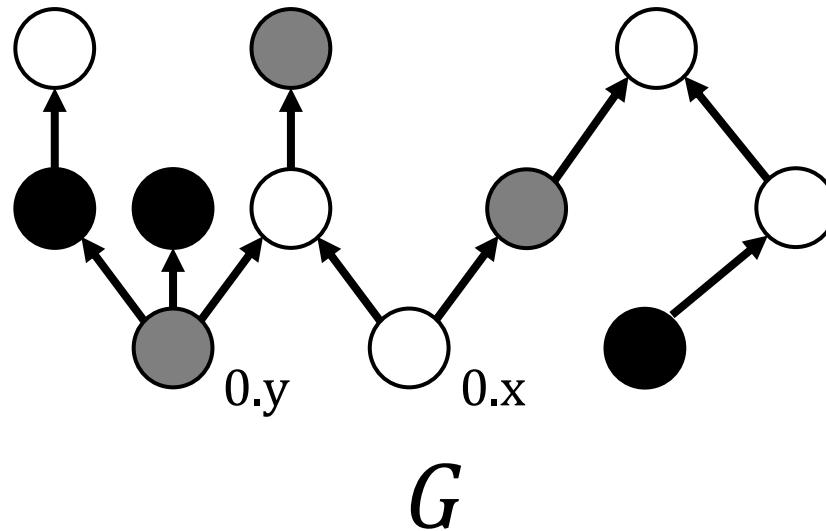
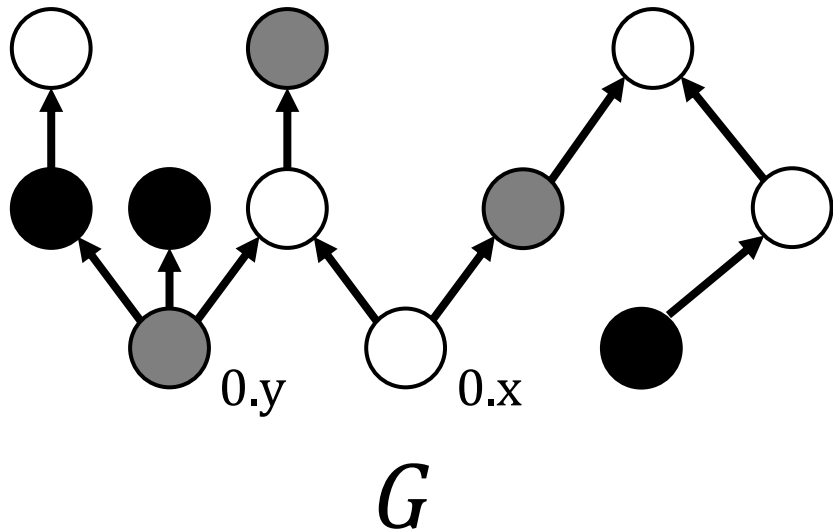
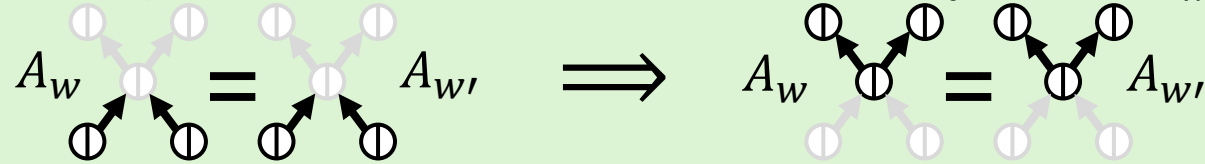


Hypothesis



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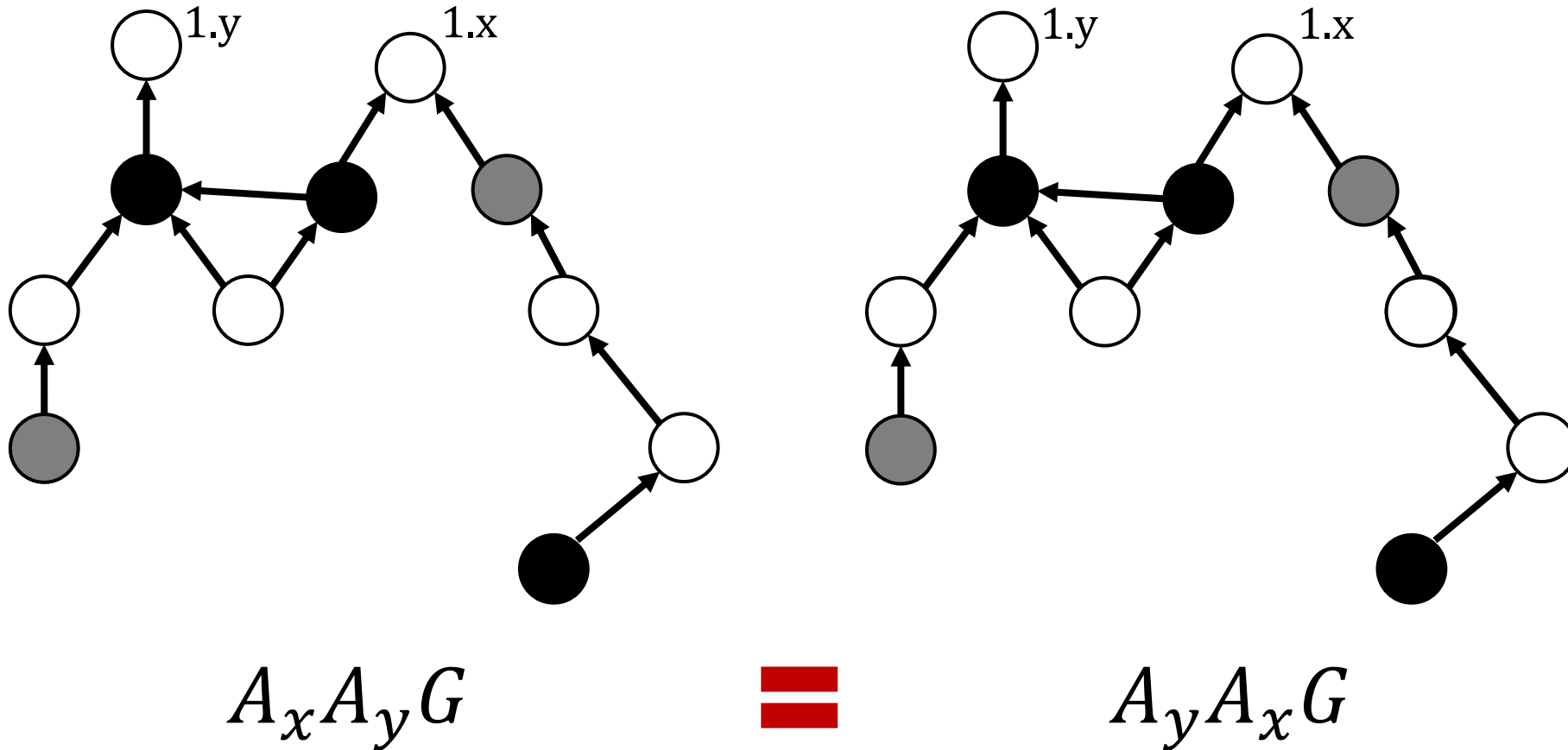
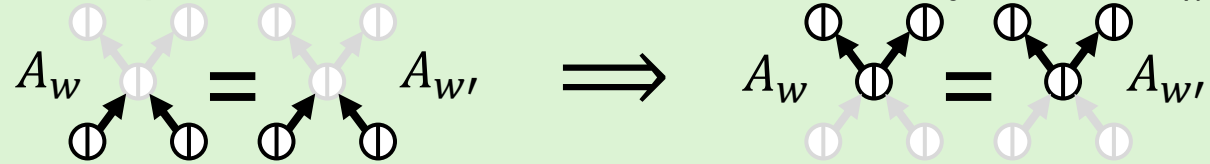


Hypothesis

1. Commutative
 $A_x A_y G = A_y A_x G$

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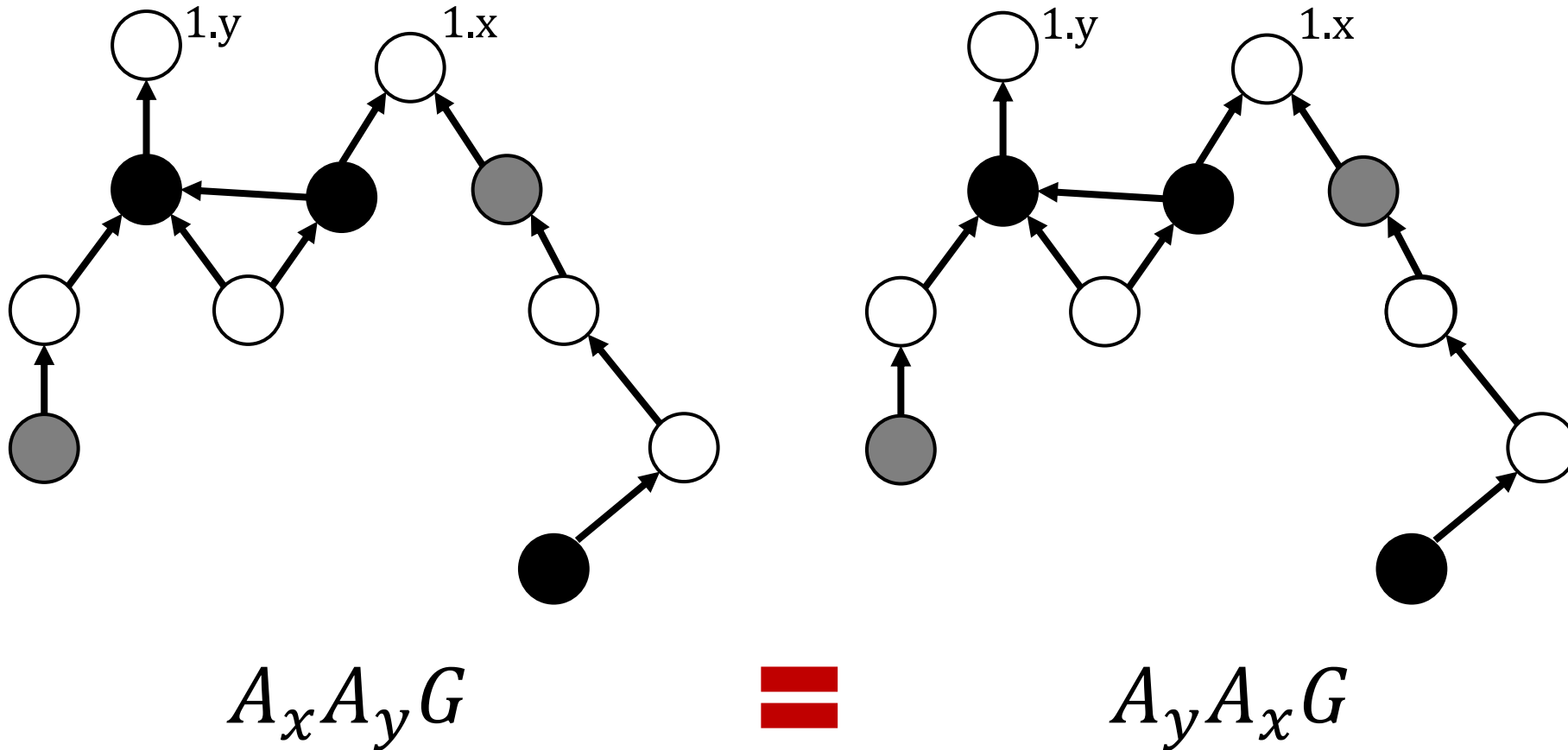
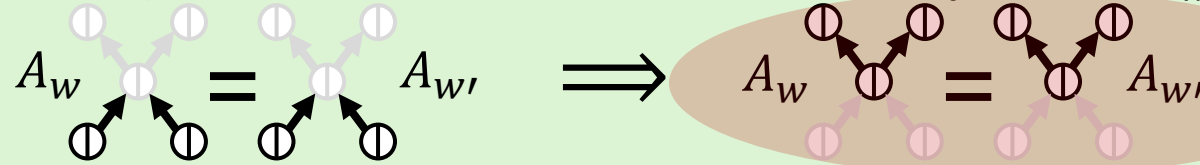


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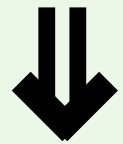
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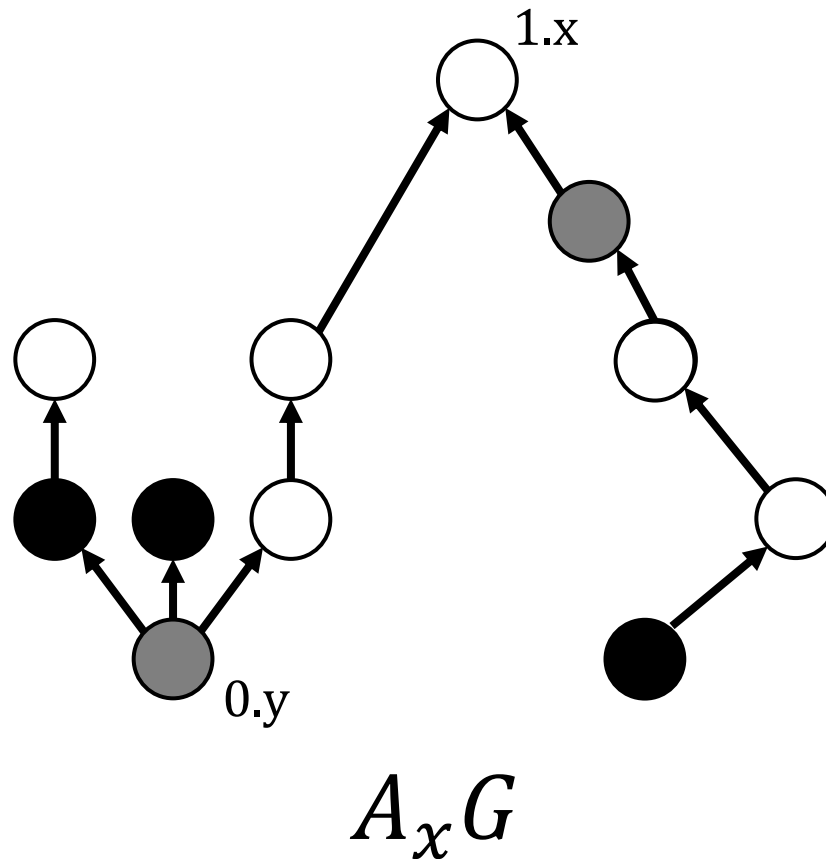
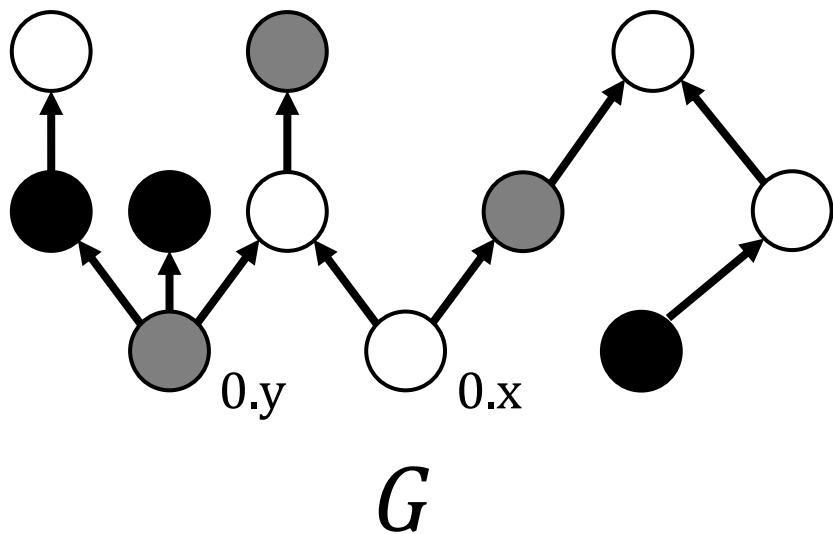
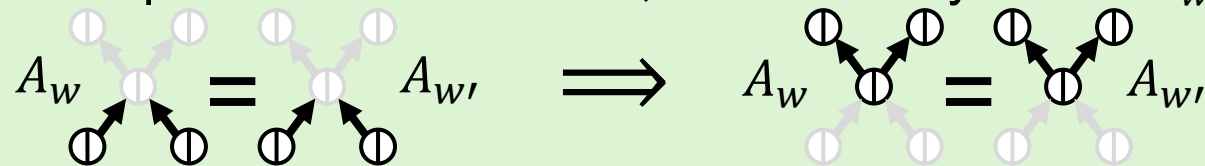


Consistent cuts

- $A_x A_y G || A_y A_x G$

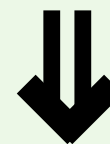
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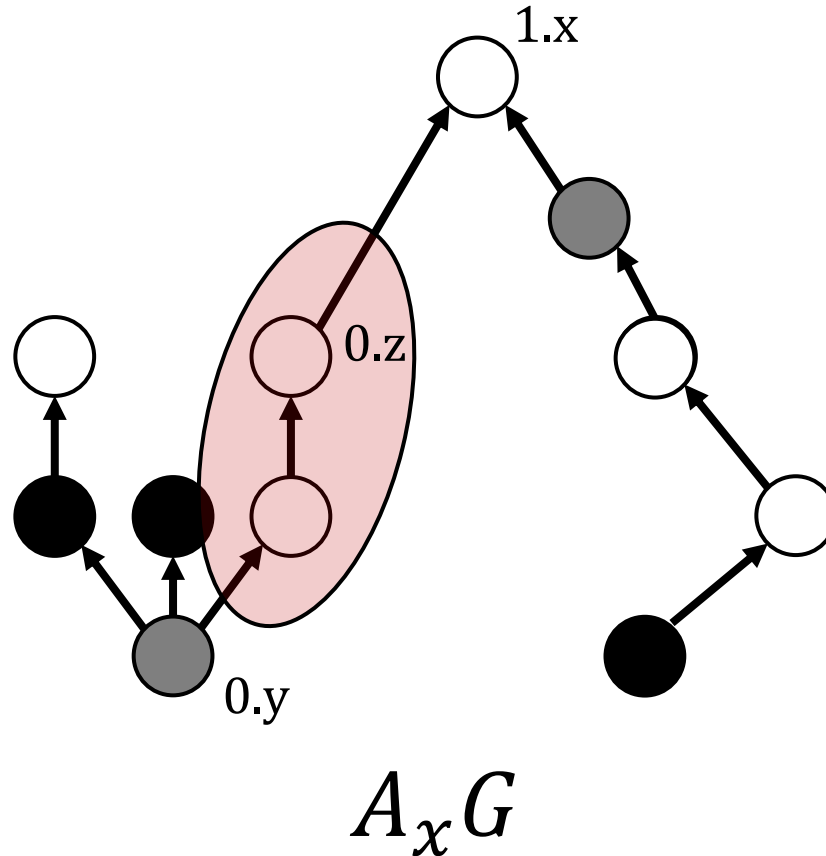
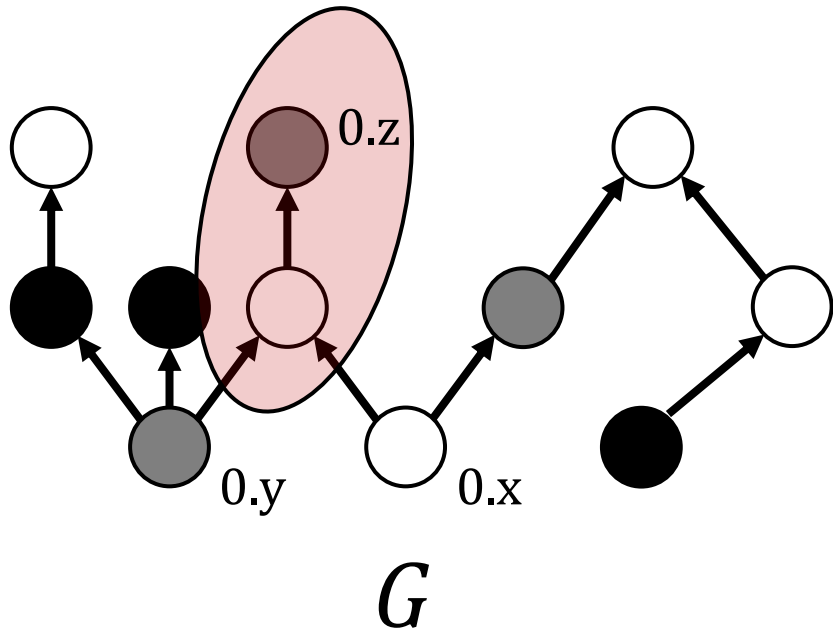
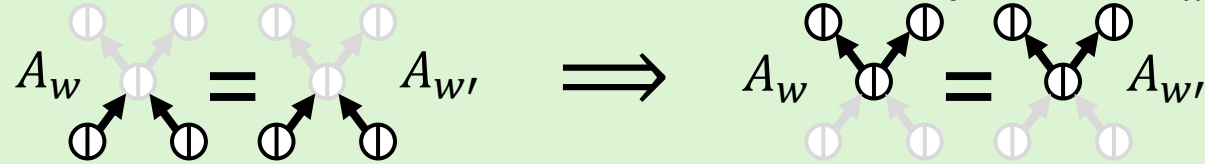


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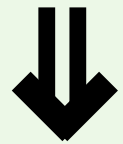
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- Commutative
 $A_x A_y G = A_y A_x G$

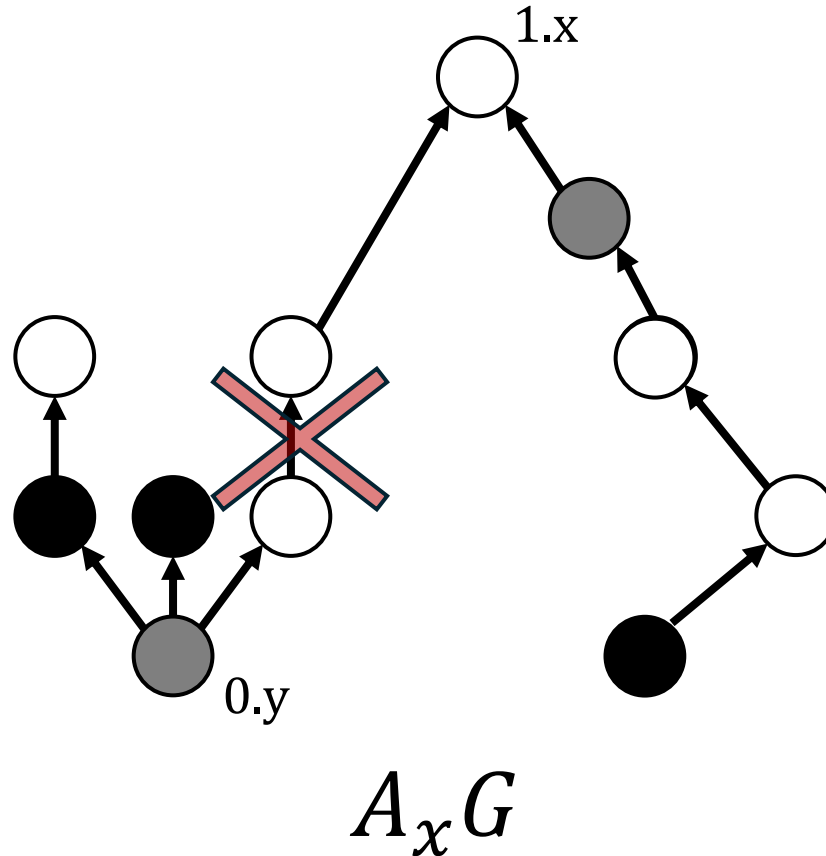
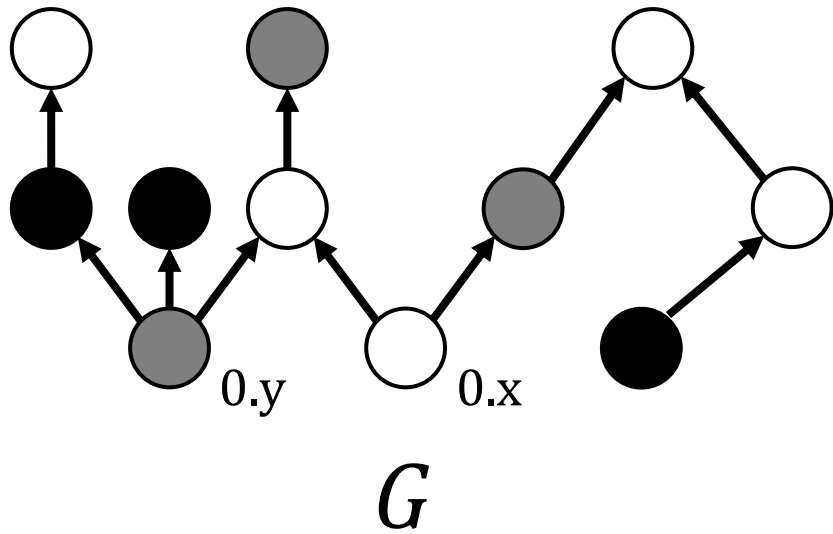
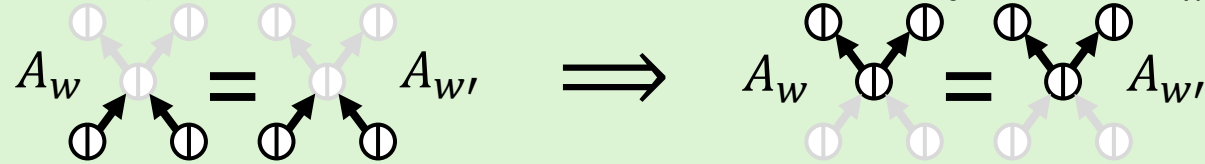


Consistent cuts

- $A_x A_y G || A_y A_x G$

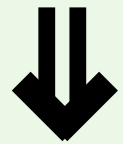
Ensuring determinism

Goal : prove A deterministic, i.e. we always have $A_w || A_{w'}$:



Hypothesis

1. Commutative
 $A_x A_y G = A_y A_x G$
2. Edge decreasing

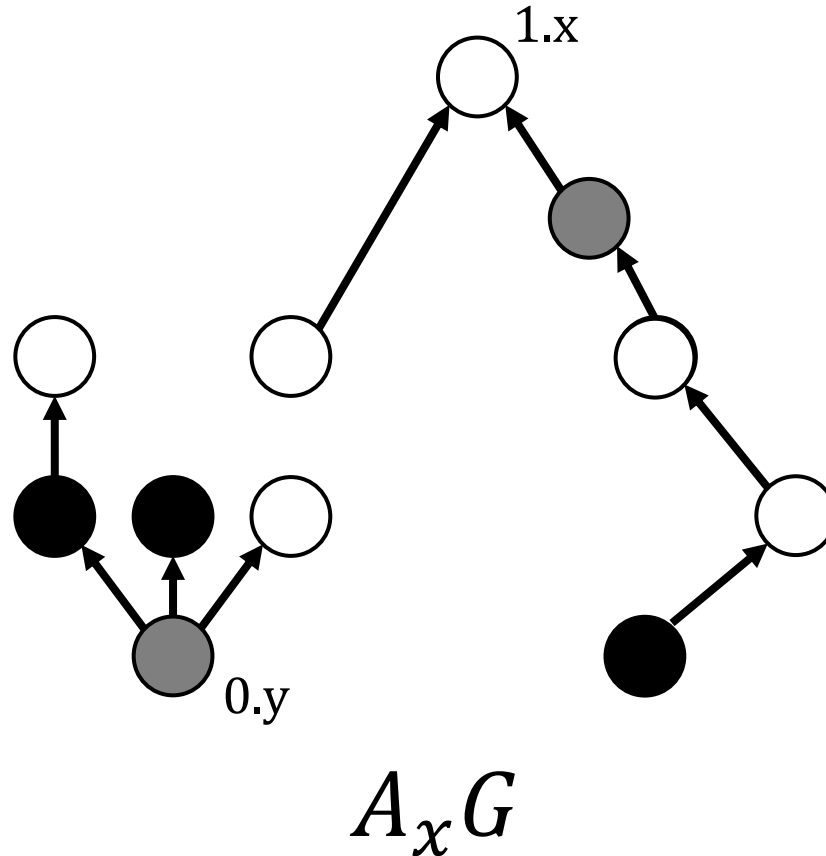
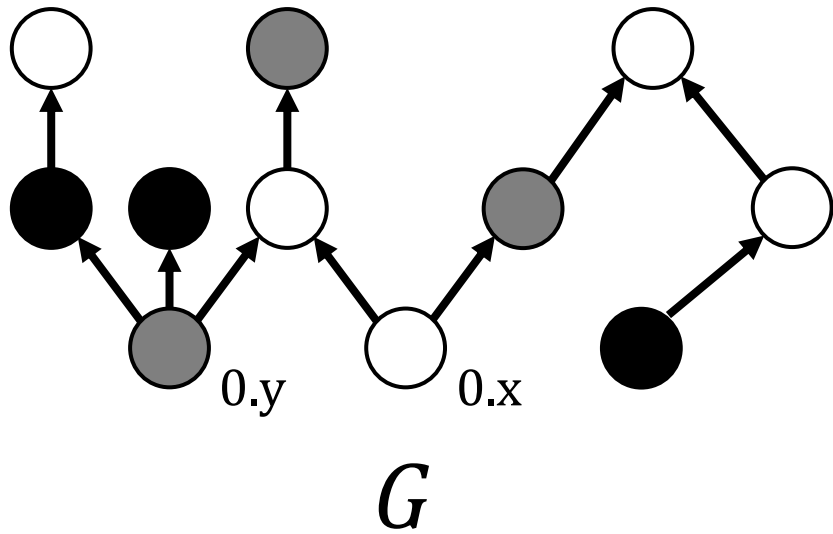
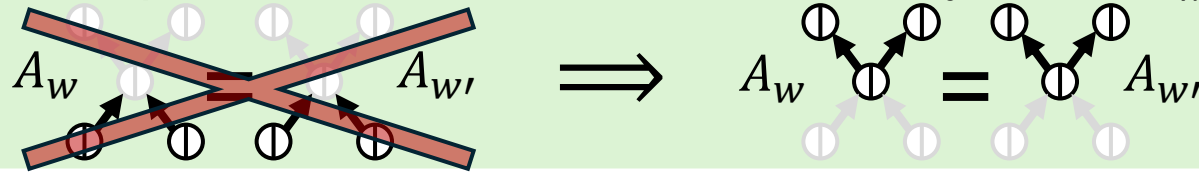


Consistent cuts

1. $A_x A_y G || A_y A_x G$

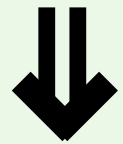
Ensuring determinism

Goal : prove A deterministic, i.e. we always have $A_w \parallel A_{w'}$:



Hypothesis

1. Commutative
 $A_x A_y G = A_y A_x G$
2. Edge decreasing

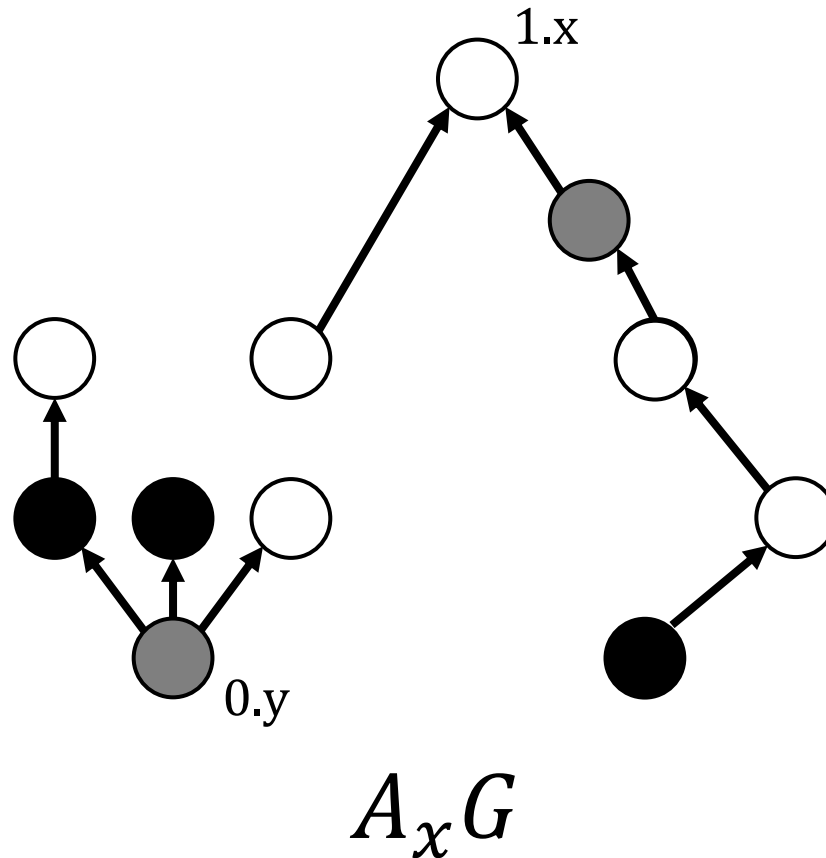
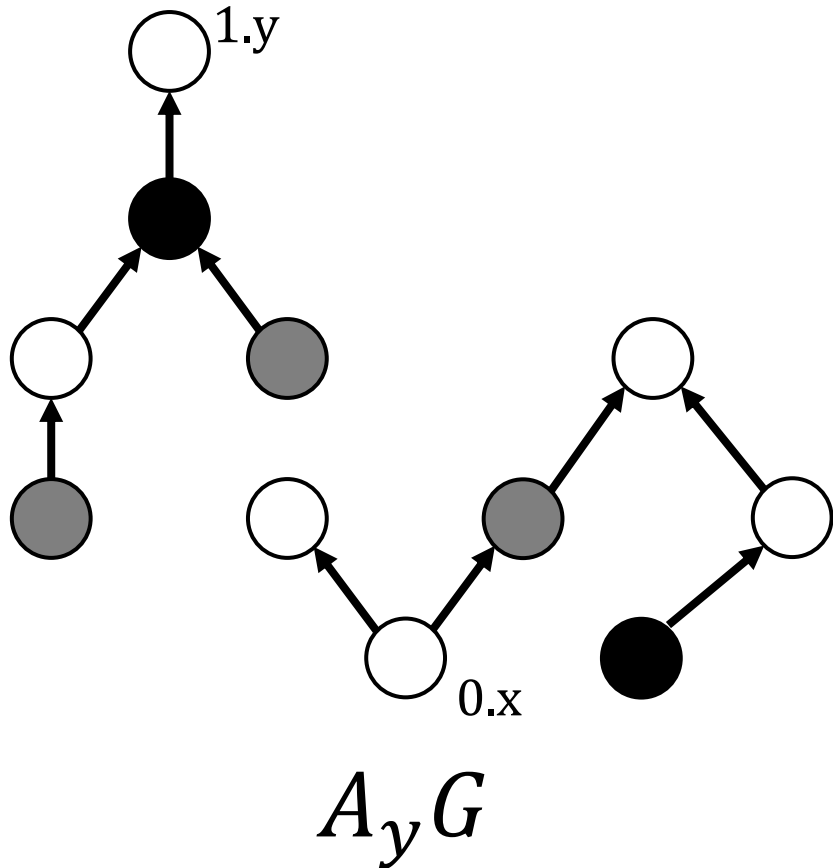
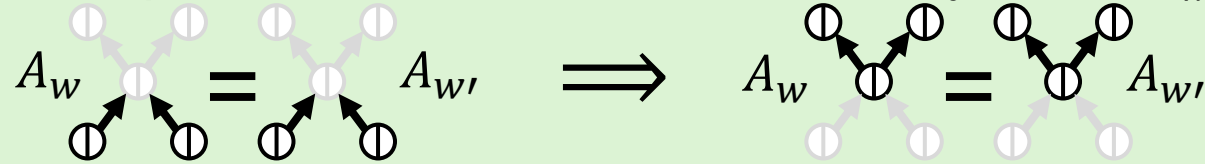


Consistent cuts

1. $A_x A_y G \parallel A_y A_x G$
2. $A_x G \parallel G$

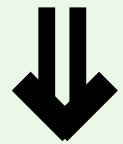
Ensuring determinism

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Hypothesis

1. Commutative
 $A_x A_y G = A_y A_x G$
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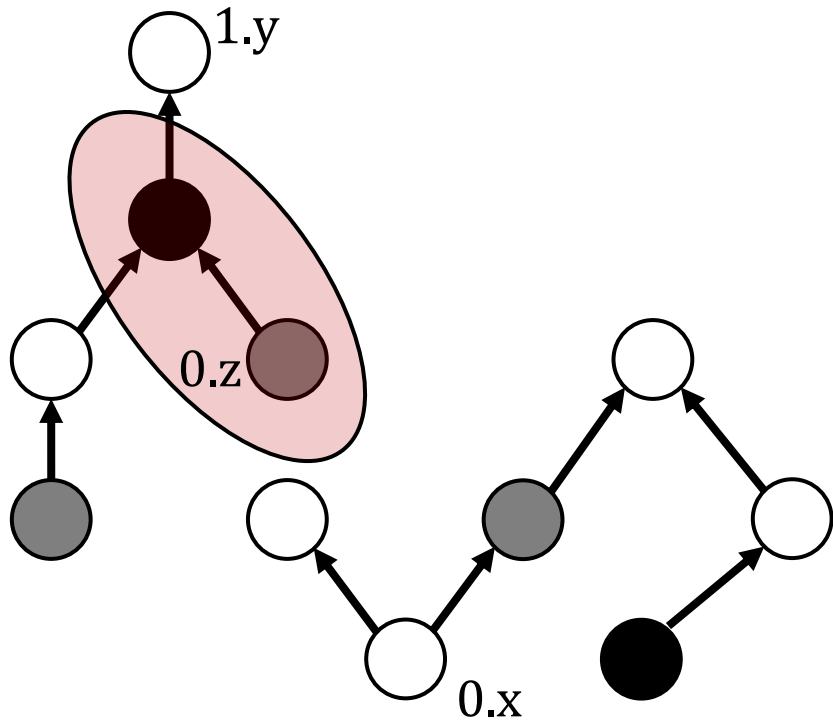
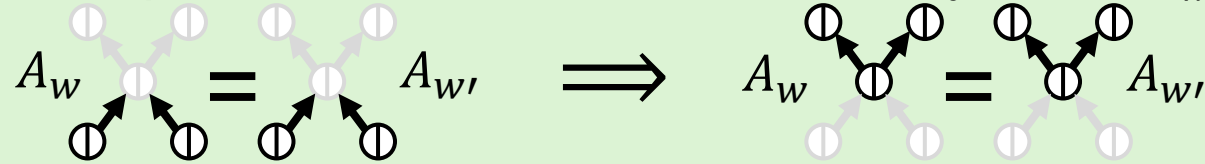


Consistent cuts

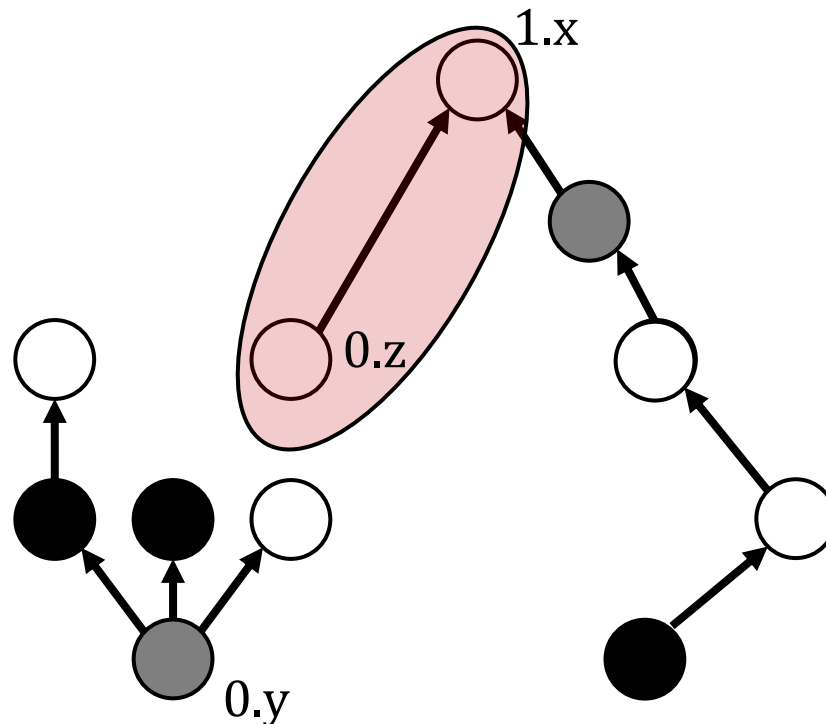
1. $A_x A_y G || A_y A_x G$
2. $A_x G || G$

Ensuring determinism

Goal : prove A deterministic, i.e. we always have $A_w || A_{w'}$:



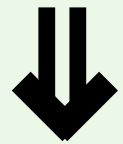
$A_y G$



$A_x G$

Hypothesis

1. Commutative
 $A_x A_y G = A_y A_x G$
2. Edge decreasing

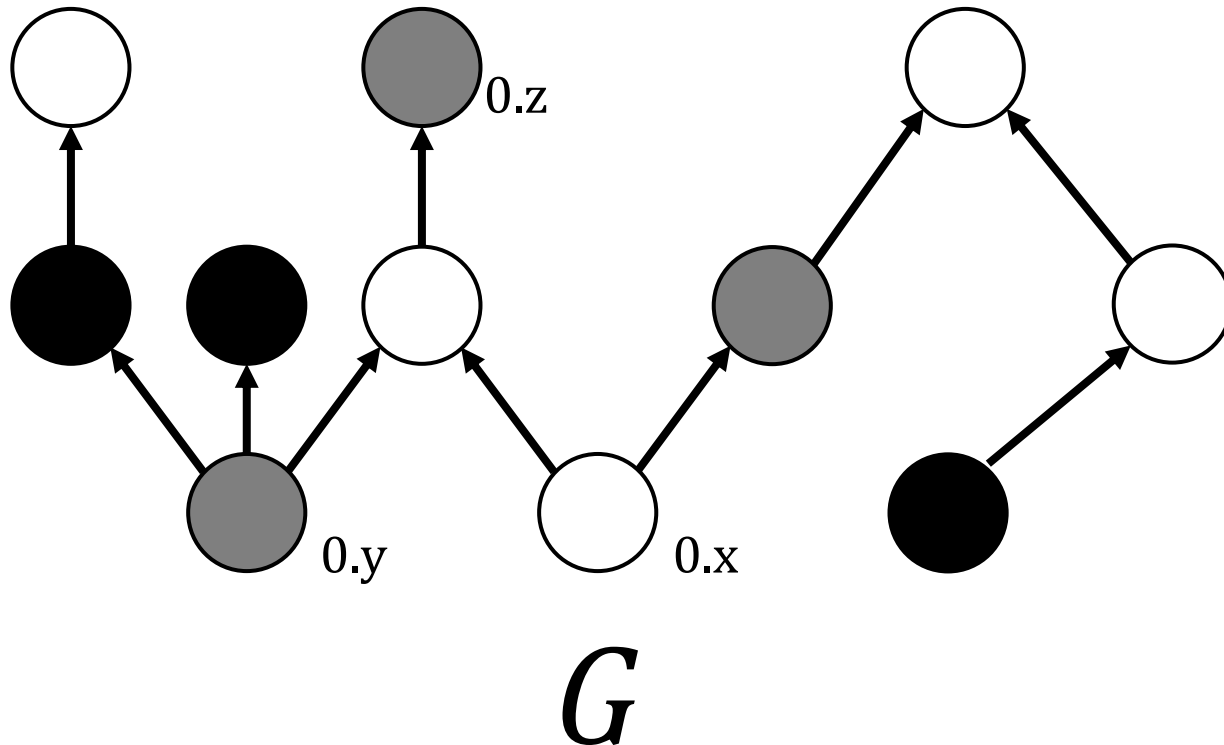
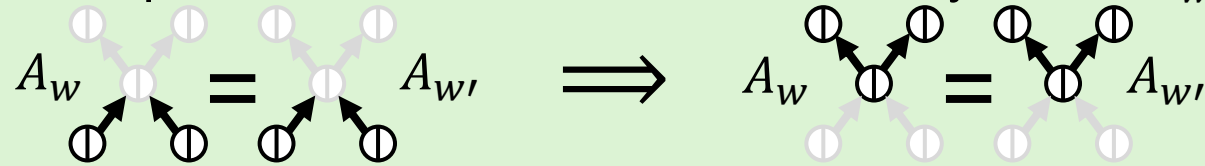


Consistent cuts

1. $A_x A_y G || A_y A_x G$
2. $A_x G || G$

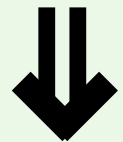
Ensuring determinism

Goal : prove A deterministic, i.e. we always have $A_w || A_{w'}$:



Hypothesis

1. Commutative
 $A_x A_y G = A_y A_x G$
2. Edge decreasing

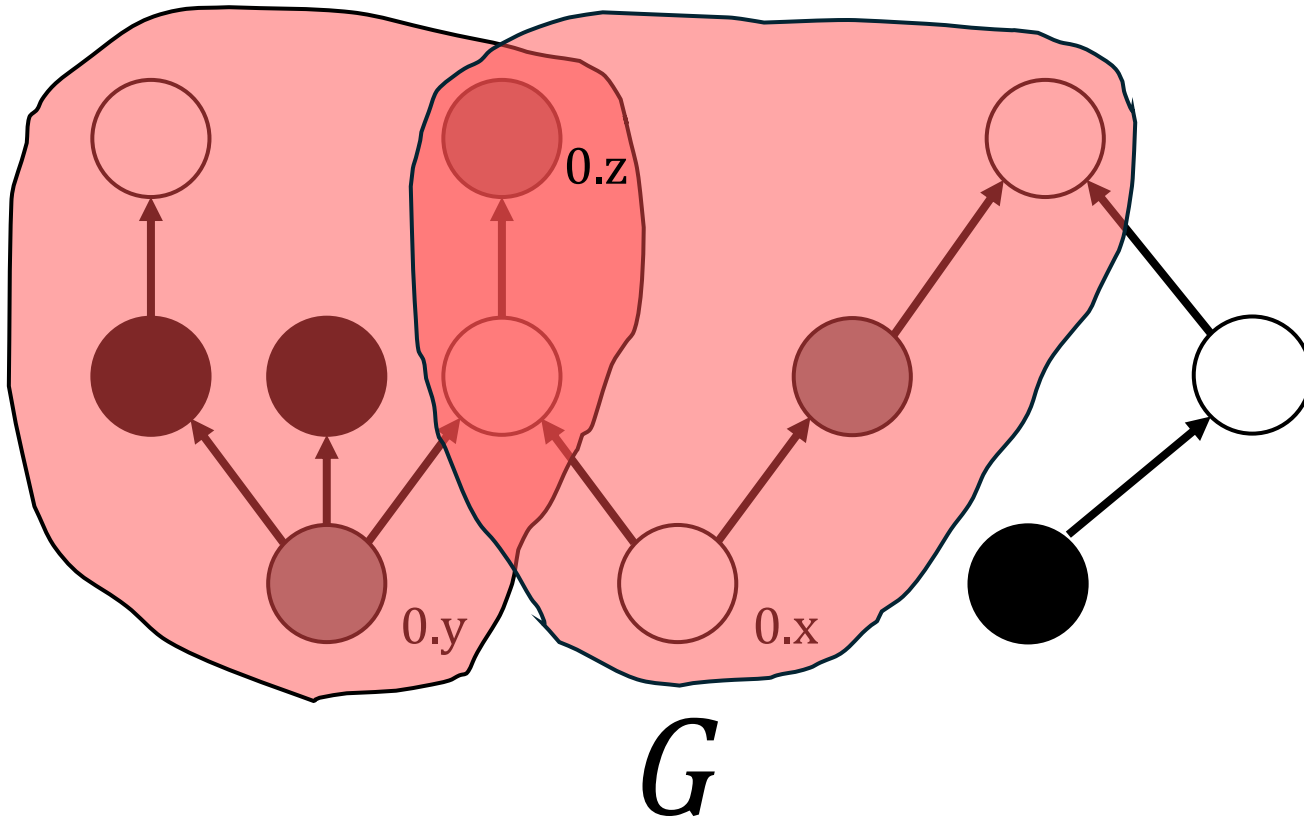
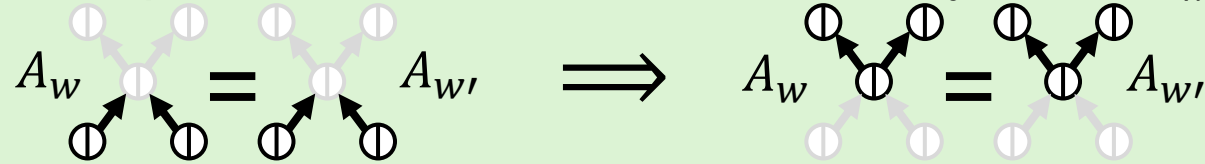


Consistent cuts

1. $A_x A_y G || A_y A_x G$
2. $A_x G || G$

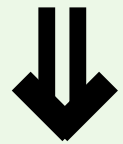
Ensuring determinism

Goal : prove A deterministic, i.e. we always have $A_w || A_{w'}$:



Hypothesis

1. Commutative
 $A_x A_y G = A_y A_x G$
2. Edge decreasing

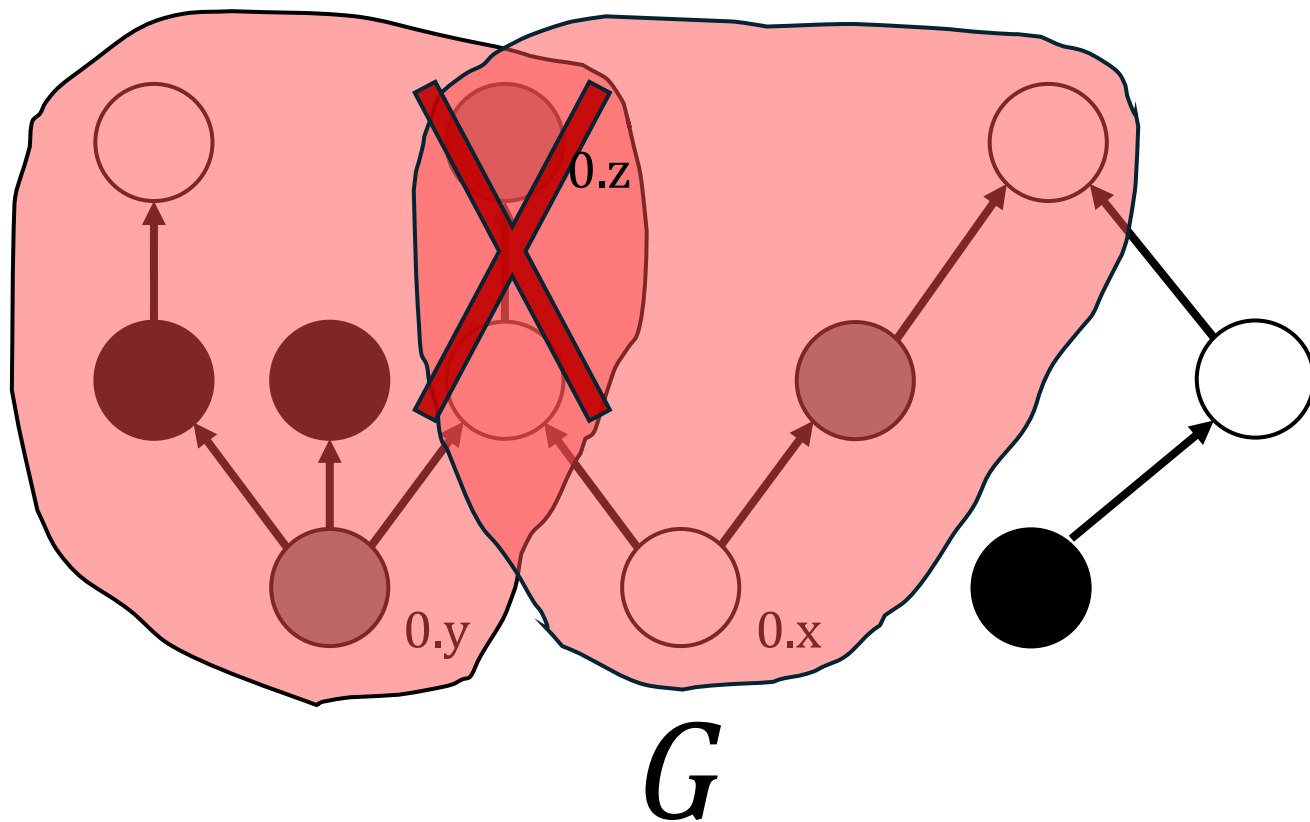
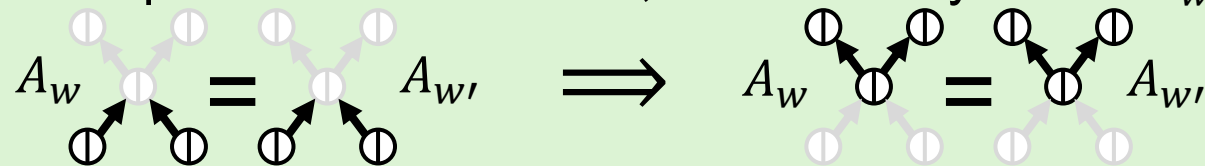


Consistent cuts

1. $A_x A_y G || A_y A_x G$
2. $A_x G || G$

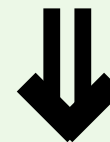
Ensuring determinism

Goal : prove A deterministic, i.e. we always have $A_w || A_{w'}$:



Hypothesis

1. Commutative
 $A_x A_y G = A_y A_x G$
2. Edge decreasing
3. Private

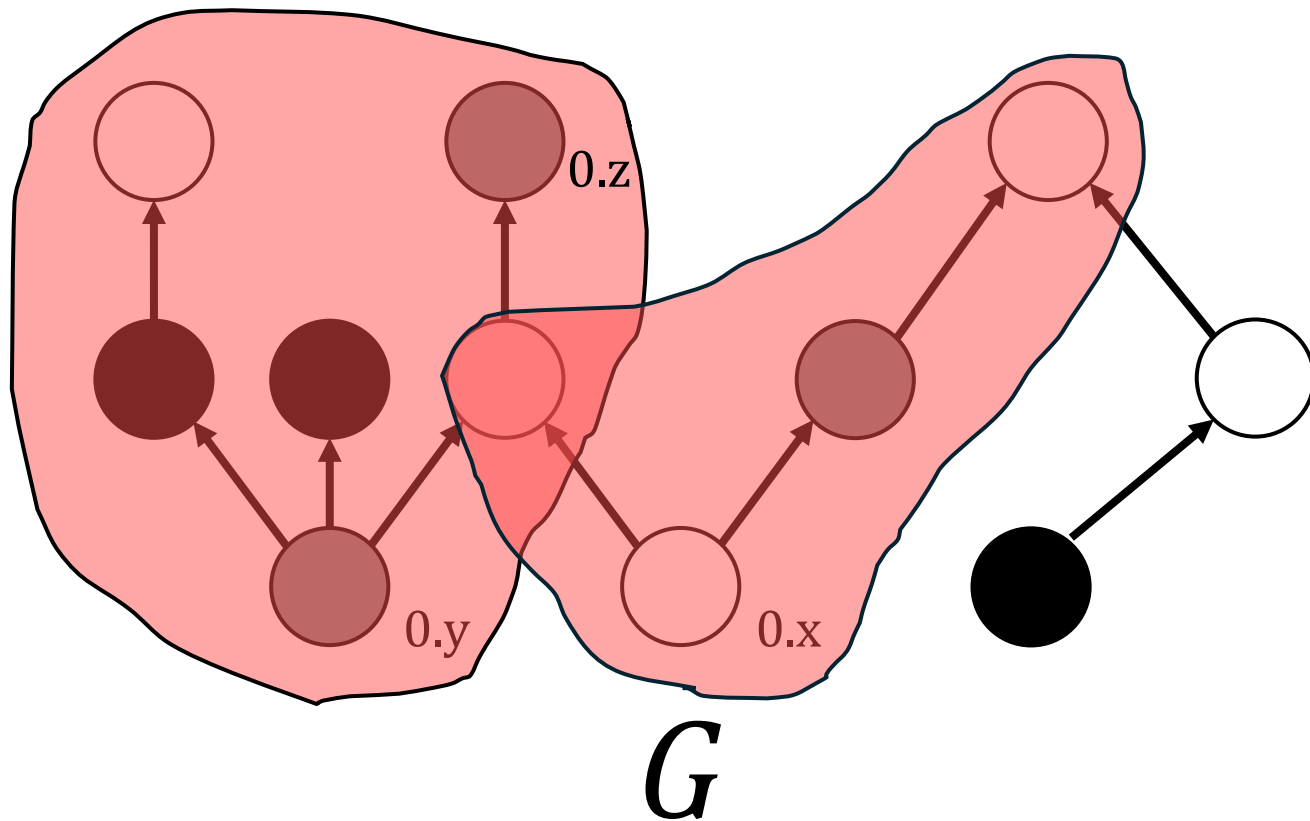
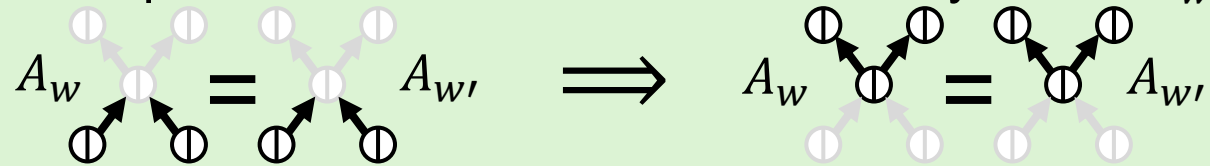


Consistent cuts

1. $A_x A_y G || A_y A_x G$
2. $A_x G || G$

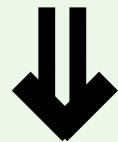
Ensuring determinism

Goal : prove A deterministic, i.e. we always have $A_w || A_{w'}$:



Hypothesis

1. Commutative
 $A_x A_y G = A_y A_x G$
2. Edge decreasing
3. Private

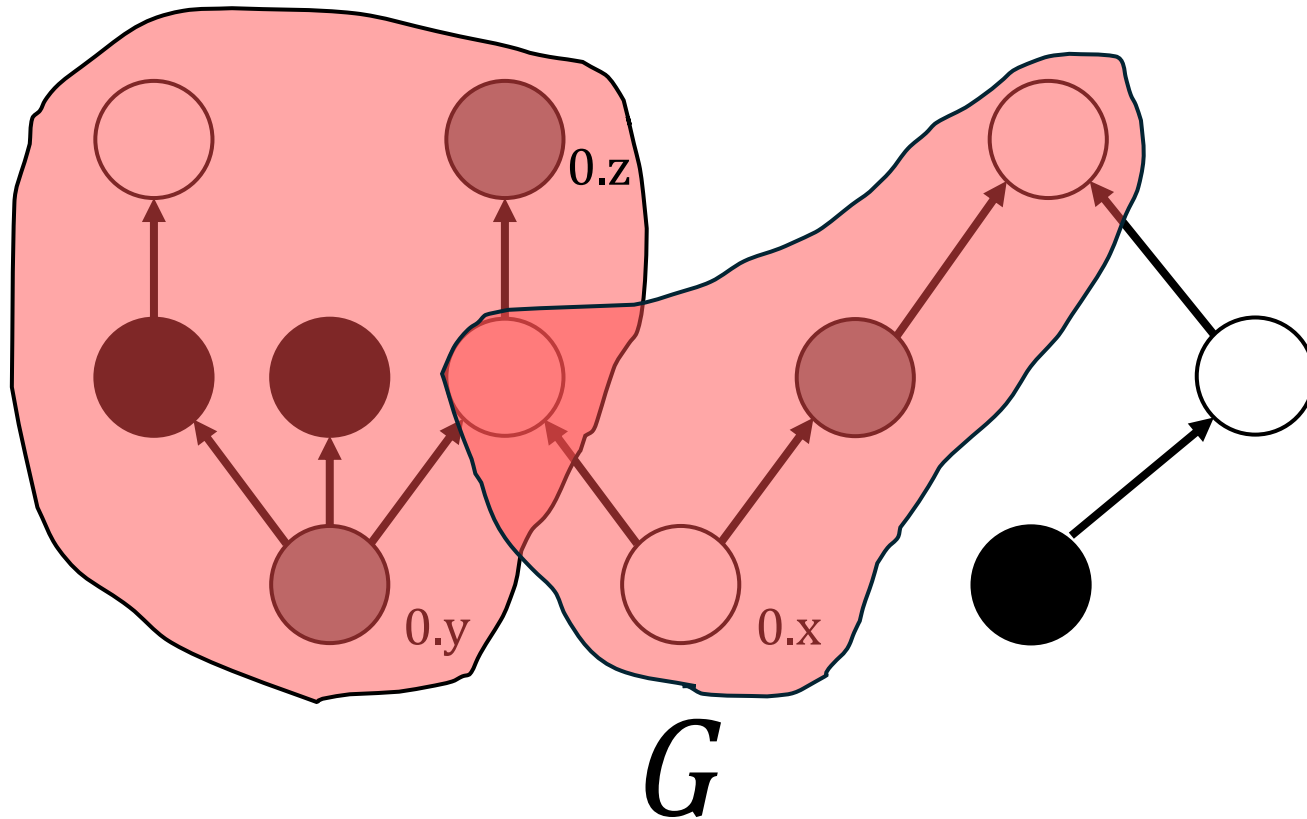
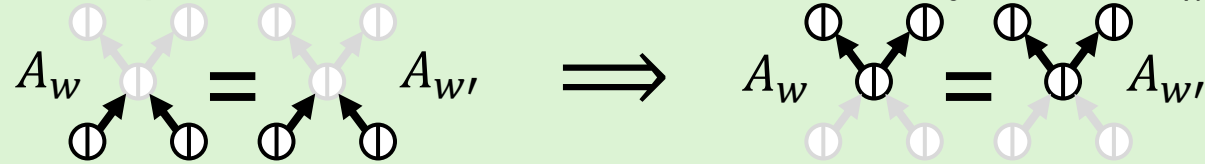


Consistent cuts

1. $A_x A_y G || A_y A_x G$
2. $A_x G || G$
3. $A_x G || A_y G$

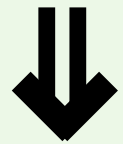
Ensuring determinism

Goal : prove A deterministic, i.e. we always have $A_w \parallel A_{w'}$:



Hypothesis

1. Commutative
 $A_x A_y G = A_y A_x G$
2. Edge decreasing
3. Private

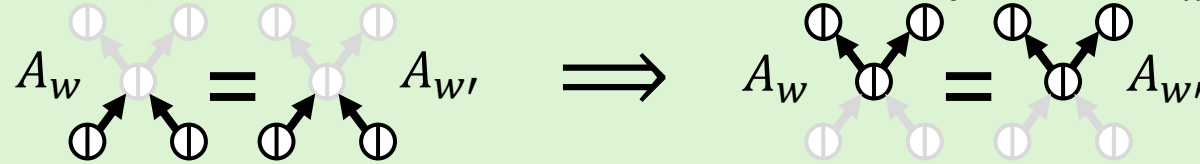


Consistent cuts

1. $A_x A_y G \parallel A_y A_x G$
2. $A_x G \parallel G$
3. $A_x G \parallel A_y G$
4. $A_w G \parallel A_{w'} G$

Ensuring determinism

Goal : prove A deterministic, i.e. we always have $A_w \parallel A_{w'}$:

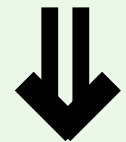


Theorem 1

Any commutative, edge decreasing and private local rule is deterministic.

Hypothesis

1. Commutative
 $A_x A_y G = A_y A_x G$
2. Edge decreasing
3. Private

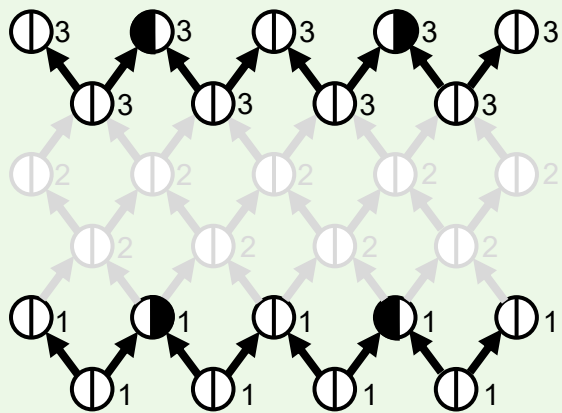


Consistent cuts

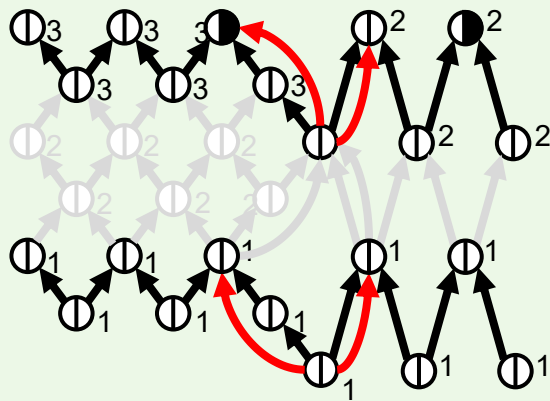
1. $A_x A_y G \parallel A_y A_x G$
2. $A_x G \parallel G$
3. $A_x G \parallel A_y G$
4. $A_w G \parallel A_{w'} G$

Conclusion

Particle system

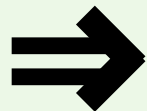


Time dilation

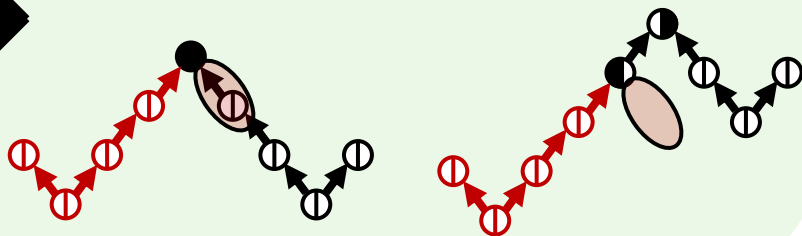


Space-time Determinism

1. Commutative
2. Edge decreasing
3. Private

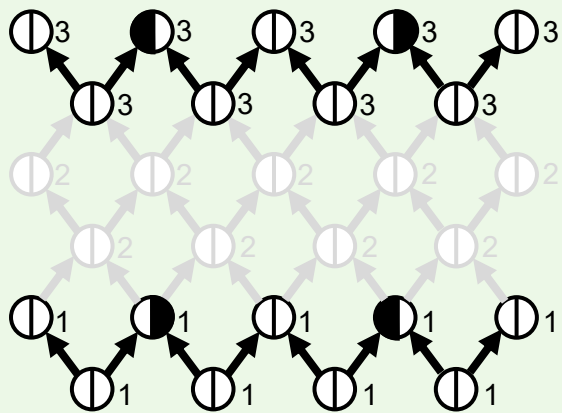


$$A_W G \quad || \quad A_{W'} G$$

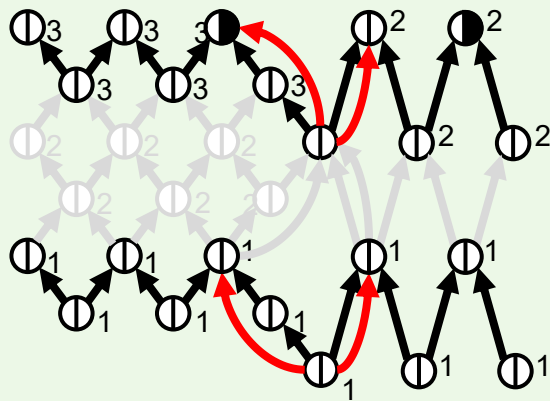


Conclusion

Particle system

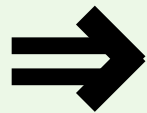


Time dilation

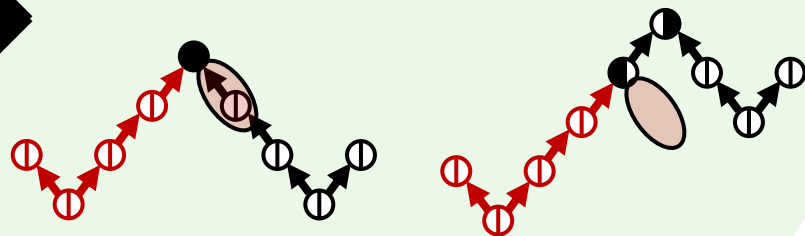


Space-time Determinism

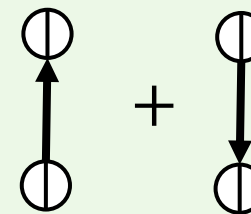
1. Commutative
2. Edge decreasing
3. Private



$$A_W G \quad || \quad A_{W'} G$$



Quantum ?



Black hole simulation ?

